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APPLICATIONS OF BAYESIAN STRUCTURAL EQUATION MODELING TO SPORT AND EXERCISE PSYCHOLOGY

Chao Ning[1], Xi An[2,*], LiLi Wu[1]
[1.] School of Exercise and Health, Shanghai University of Sports, Shanghai 200438, China;
[2.] School of Psychology, Shanghai University of Sports, Shanghai 200438, China.

Abstract: To introduce the characteristics and application methods of Bayesian structural equation model. Firstly, the advantages of the Bayesian structural equation model were discussed, and then the second-order confirmatory factor analysis was carried out using the maximum likelihood estimation and Bayesian estimation with the evaluation data of the athlete training state detection scale (32 × 7). The Bayesian estimation model incorporating small variance prior information such as cross loadings and residual correlations fit well, while the model using maximum likelihood estimation did not fit well. Analyze the reasons for the above differences, and summarize the advantages and disadvantages of the Bayesian structural equation model.
Keywords: Bayesian method; Structural equation model; Confirmatory factor analysis; Cross loading; Residual correlation

Confirmatory factor analysis (CFA) and structural equation modeling (structure equation) are often used in sports and exercise psychology research. equation model, SEM), and the maximum likelihood estimation (MLE) is the most used model parameter estimation method [1]. Due to the restriction that cross-loadings and residual correlations must be zero, this method often encounters problems such as poor model fitting [2], biased estimation of factor loadings and factor correlations [3] and so on. Under such strict constraints, the hypothetical model is often rejected[4], even if it is no longer rejected after a series of model revisions based on the revision index, it still has to face the question of capitalization on chance[5]. MUTHEN et al. [6] introduced a Bayesian structural equation modeling analysis technology (Bayesian structural equation modeling, BSEM), which can help researchers effectively deal with the above problems. This study aims to introduce the basic principles and characteristics of BSEM, and compare the differences between BSEM and maximum likelihood estimation structural equation modeling through an empirical study in the field of sport and exercise psychology.

1. INTRODUCTION TO BSEM

BSEM is an analysis technique that uses small variance information to reflect the researcher's theoretical conception and prior belief. It is the specific application of Bayesian data analysis method in structural equation model analysis. According to the viewpoint of ASP-AROUHOV et al. [7], according to the theoretical conception and prior belief, researchers can set the information prior of small variance for the non-free estimation parameters (such as cross loading and residual correlation) in the measurement model, while Instead of fixing it to 0. In other words, BSEM uses information priors with small prior variances to replace zero-cross loads and zero-residual correlations, removing unnecessary and strict model constraints, so that the hypothetical model can better reflect the researcher's theoretical ideas and priors. belief. In classical statistical methods, releasing these parameters for free estimation often leads to unidentifiable models, while Bayesian methods can use small variance priors to ensure that models can be identified. BSEM can set prior information of small variance for all non-free estimated parameters in the model. This study only involves two types of parameters related to cross load and residual in the measurement model, and the prior setting of some non-free estimated parameters in the structural model is not discussed. within range.

1.1 Information Prior for Cross Loading

For researchers who are accustomed to using maximum likelihood estimation, the practice of setting cross-loading prior information may be uncomfortable. In the process of using maximum likelihood estimation for CFA or structural equation model analysis, it is usually assumed that a measurement item is only affected by a single latent factor, and the cross load is fixed at 0. From the Bayesian point of view, the zero-crossing load can actually be regarded as a prior distribution with both mean and variance being 0. In fact, the cross loading is unlikely to be exactly 0, but obeys a normal distribution with mean 0 and small variance. Therefore, setting a normal prior distribution with zero mean and small variance for the cross load can more properly reflect the theoretical concept and prior belief of the researcher. For example: setting the cross load $\lambda \sim N$ (0, 0. 01) means that the cross load has a 95% probability of -0. 2 ~ 0. 2. A cross loading of 0.2 obtained using standardized data can be considered a small loading. In other words, this means that the researcher's prior belief in the cross loading is that the cross loading is very close to zero, not exactly zero. Generally speaking, the smaller the variance, the greater the amount of information carried by the prior, which means that the researchers are more confident that the mean value of the cross load is 0.
When using maximum likelihood estimation, releasing all cross loadings in the measurement model so that they can be estimated freely can lead to model identification problems. When Bayesian estimation is used, the prior distribution of

cross loads can be obtained by setting small variance information priors for all cross loads and combining the information about cross loads in the observed data, thereby avoiding the problem of model identification. The prior setting of small variance information will not make the cross loading estimate deviate too far from the prior mean, which will affect the acceptability of the posterior predictive pvalue (PPp). In general, the larger the variance, the greater the probability that the cross loadings deviate farther from the prior mean. If the prior variance is 0.08, the corresponding 95% confidence interval is -0.55 ~ 0.55. More importantly, the larger the variance, the less information the prior distribution carries, the more likely it is to encounter model identification problems, and even the MCMC iterative process cannot converge.

1.2 Informative Prior on Residuals

In the classical statistical method, the principle of independence of the residuals needs to be satisfied, so it is assumed that the correlation between the residuals of the measurement items in the CFA and the structural equation model is 0. In fact, the purpose of factor analysis is to use a small number of latent factors to represent a large number of measurement items. During this process, some secondary factors are omitted from the model. From this, it is not difficult to imagine that the residuals of some measurement items may have a certain degree of correlation, but it is more difficult to clearly find out which residuals have a covariate relationship. While using maximum likelihood estimation, releasing all residual correlations would render the model unrecognizable by exhausting all degrees of freedom, BSEM provides an alternative for faithfully representing residual correlations in the model.

BSEM uses a residual covariance matrix with elements in the lower triangular region instead of the residual covariance matrix with only diagonal region elements in classical structural equation models. It is not difficult to understand that the residual covariance matrix of BSEM contains two parts, one part is the residual variance on the diagonal,

The other part is the residual correlation in the lower triangular region.

In BSEM, the inverse Wishart distribution (inverse Wishart distribution) sets the prior distribution of the residual covariance, and then obtains its posterior distribution. By setting a small variance prior distribution for the residual covariance, the residual variance and residual correlation can be constrained within a smaller value range. That is, BSEM allows a small deviation of the residual covariance from its mean. Generally speaking, the residual correlation and residual variance need to set the prior distribution separately, and the prior setting of the residual correlation must be strict enough, so as to ensure that the important correlation between items is through factor loading, latent variable correlation and other channels Reflected.

1.3 Features of Bsem

MLE is the most commonly used estimation method for structural equation models, while Bayesian estimation provides a more flexible alternative estimation method for structural equation models [6]. Compared with the maximum likelihood estimation structural equation model, BSEM can use the Gibbs sampling algorithm to incorporate prior information related to cross loads and residuals into the analysis process, so that the hypothetical model can better reflect the researcher's theoretical ideas and prior beliefs [6]. Specifically: 1) With the help of Gibbs sampling algorithm, Bayesian estimation can still obtain unbiased estimation of model parameters under the condition of relatively small sample size. The use of maximum likelihood estimation requires a larger sample size, such as at least 200 or more [1]. 2) The application of BSEM can fully consider the previous research results and incorporate them into the current research in the form of prior distribution, which reflects the inheritance of research. Specifically, BSEM can use the prior information on the direction and strength of variable relationships obtained from meta-analysis or existing theories, combined with current observation data, to construct credible intervals or posterior predictive intervals of parameters. interval , PPI), so as to update prior knowledge [8]. 3) The use of maximum likelihood estimation needs to satisfy the multivariate normal distribution. In general, the more complex the model, the more difficult it is to satisfy a multivariate normal distribution. Bayesian methods are less concerned with satisfying the multivariate normal distribution. 4) The use of maximum likelihood estimation generally does not allow the existence of missing values, and it is necessary to imput the missing values or delete the data with missing values. According to Bayesian statistics, missing values, latent variables and parameters have the same properties [9], so missing values can be regarded as unknown parameters and included in the model for analysis. 5) Generally speaking, BSEM has more advantages in dealing with complex models. For models that are poorly fitted by MLE, it may be possible to obtain an acceptable fit by applying Bayesian estimation. In addition, for models with insufficient recognition, by incorporating prior information, the posterior distribution of parameters can still be obtained with the help of Gibbs sampling algorithm, which cannot be achieved in maximum likelihood estimation structural equation models [10]. 6) The credible intervals provided by BSEM for each parameter are more intuitive to interpret. Confidence intervals in classical statistical methods (confidence interval, CI) is often misinterpreted as the probability that the parameter lies in a particular interval. In fact, CI does not reflect the attributes of parameters, but more reflects the attributes of procedures to obtain CI [11]. Taking 95% CI as an example, its accurate interpretation should be: 95% of the estimated values generated by infinite sampling from the same population are located in this interval [8]. The CI provided by the Bayesian statistical method means: under the condition of given observation data, the probability that the true value of the parameter is located in a specific interval [8]. 7) Using BSEM, researchers can obtain the probability of null hypothesis or alternative hypothesis under the given observation data conditions [12]. The p-value in classical statistics does not provide this information. Its definition of pvalue is: when the null hypothesis is true, the probability of obtaining the observed data or more extreme data [13]. In the Bayesian method, researchers can calculate the posterior probability of hypothesis or model parameters based on observed data and prior distribution.

2. CASE STUDY

How to prevent overtraining is an important research topic in the field of sports science. Excessive training will not only affect athletes' training and competition, but also make fitness exercise participants lose their enthusiasm for participating in physical exercise, or even give up physical exercise completely [14]. According to the principle that prevention is better than cure, it is very important to systematically monitor the training process of athletes and adjust the exercise load in time. The Athlete Training Status Monitoring Scale[15] is a set of simple psychological measurement tools used to monitor the training status, which can help coaches and sports researchers understand the training status of athletes and detect overtraining symptoms in time. The training state refers to the athlete's
The physical and psychological conditions during training include two aspects: stress and recovery. Stress is an individual's physiological and psychological response to external stimuli, and recovery is a time-dimension, individual or inter-individual, multi-level (such as physiological The reconstruction process of operational ability [16]. For the convenience of daily monitoring, the author developed 3 simplified scales at the same time, which can be converted into full scale scores through regression equations. This set of evaluation tools meets the needs of daily monitoring and scientific research on athletes' training status [17-22].
This study demonstrates the use of Bayesian confirmatory factor analysis by examining the psychometric characteristics of its simplified version, the Athlete Training Condition Monitoring Scale 32 × 7. At the same time, by comparing with the results of MLE confirmatory factor analysis, the differences between the two estimation methods in parameter estimation and model fitting are demonstrated.
twenty one method

2.1 Research Object

This study takes high-level college athletes as the research object. With the consent of the coaches, the organization will fill in the questionnaire after the training. A total of 263 college athletes completed the "Athlete Training Status Monitoring Scale 32 × 7". Among them, first-level and above athletes
67 people, 196 second-level athletes; 107 women. Age is x x ± s = 20. 06 ± 1. 24. There are both individual and team sports. In particular, according to HAIR et al. [1], when the sample size is more than 300 and each construct contains at least 3 measurement items, the violation of estimation can be effectively avoided. For illustration purposes, a sample size lower than the rule of thumb was chosen for this study case.

2.2 Assessment Tool

The measurement tool in this study is "Athlete Training Status Monitoring Scale 32 × 7", with a total of 32 items. The subjects make choices on the 7-level Likert scale. The larger the score, the higher the frequency of the situation described in the item higher. The scale contains 2 second-order factors of stress and recovery, in which stress is measured by 4 first-order factors of emotional stress, fatigue, mental exhaustion and mental fatigue; recovery is measured by 4 first-order factors of feeling good, self-regulation, self-efficacy and physical recovery. A first-order factor measure. Each factor is measured by 4 items respectively.

2.3 Data Analysis

The data analysis method is mainly CFA, with the help of Mplus 7. 4 [23], two methods of maximum likelihood estimation and Bayesian estimation are used for statistical inference. CFA is first performed using maximum likelihood estimation. At this point, each entry is only allowed to be assigned to one construct, and the residual correlation is zero. $\chi2$/df, CFI, TLI, RMSEA, SRMR and other indicators were used to evaluate the model fitting degree. Generally speaking, $\chi2$/df is less than 3, CFI and TLI are greater than 0.9, and RMSEA and SRMR are less than 0.05, indicating that the model fits well[1].
The steps of Bayesian confirmatory factor analysis:
1) Model setting. In order to facilitate the use of standardized values for the prior information of parameters such as factor loadings, the scores of all observation items are first standardized in the model setting stage. The model specification uses the fixed variance method instead of the fixed load method. Then, on the basis of the classic second-order measurement model, the cross loading and residual correlation of the first-order measurement model are added.
2) Selection and setting of prior information. Information priors Set parameter prior distributions with the model prior command. According to the recommended standard of standardized factor loading by HAIR et al. [1], and the report of Yan Ning [15] on the factor loading of the scale, the prior mean value of the factor loading is determined to be 0.7. According to the suggestion of ASPAROUHOV et al. [7], Choosing 0. 02 as the prior variance of the factor loading means that the factor loading is likely to be 0. 42 ~ 0. 98. The prior setting of the path coefficient between the second-order factor and the first-order factor of the scale is consistent with the factor loading. The setting of the prior information of the cross load is a normal distribution with a mean of 0 and a variance of 0. 02, which means that the

cross load is likely to be between ± 0. 28. Due to the use of standardized data and fixed variance method for analysis, ± 0.28 can be considered as a small cross loading.

According to the suggestion of MUTHEN et al. [6], the prior distribution related to the residuals of the measurement items is set as IW (0, 38), which obeys the inverse Wishart distribution (inverse Wishart distribution) ; The prior distribution of residual variance is set as IW (1, 38), which obeys the inverse Wishart distribution with prior mean value of 0.2 and prior variance of 0.027. According to MUTHEN et al. [6] As suggested by , fixed Bayesian iterations 50 000 times. The rest adopt the default settings of Mplus.

3) Parameter estimation. Use the potential scale reduction (PSR) indicator to evaluate the convergence process. The core idea is: when the variance between chains is less than the variance within the chain, it means that the results between the chains tend to be consistent, which means that the iterative process converges up. When the PSR is less than 1.1, it can be considered to be converged [9]. In addition, it is necessary to combine graphic indicators such as trace plots to make judgments. The converged trace plots show rapid up and down changes, and there is no long-term trend or arbitrary drift. The Bayesian estimation procedure yields 95 % CIs for each parameter by which the parameter estimates can be interpreted. If the 95 % CI does not contain 0, the parameter can be considered unlikely to be [10]

4) Posterior predictive check. Its core idea is that when the model fits well, the difference between the simulated data and the sample data should be very small [9], which is essentially a prediction accuracy.

way to assess the quality of the model. Usually PPp value and chi-square difference value 95% posterior prediction interval (Δχ2 95% PPI) to judge. The PPp value reflects the ratio of the number of times that the observed data is more representative of the model to be estimated than the simulated data, and its reasonable range is 0. 05 ~ 0. 95. When the PPp value is near 0. 50 and Δχ2 95% PPI is basically symmetrical around 0, which means that the model fits well; if the PPp value tends to 0 or 1, it means that the hypothetical model does not fit well with the observed data [9]. In practice, if the PPp value is 0. 30 ~ 0. 70, it is assumed that the model fit is acceptable [24].

5) Sensitivity analysis. That is, by setting different prior distributions, comparing model parameters under different prior information conditions

Estimate the difference in results to test how much the choice of prior distribution affects the posterior distribution. If the model parameter estimation results under different prior information conditions are not much different, it shows that the choice of prior distribution has little influence on the posterior distribution, which can support the rationality of prior information selection to a certain extent [9]. This example is carried out under 3 different priori conditions: no information priori, factor loading and cross loading prior information, and factor loading, cross loading and residual related prior information [6, 25]. The uninformative prior model estimation adopts the default setting of Mplus, such as: the default prior distribution of the measurement item mean and factor loading is N (0, 1010), which means that the prior distribution of these parameters in the model obeys the mean value of 0 and the variance is infinite. normal distribution. The prior distribution of the residual variance of the measurement items is IG (-1, 0), which obeys the inverse gamma distribution with prior mean and infinite prior variance. The deviance information criterion (DIC) is used for model comparison, and the smaller the DIC value, the better the model fit [9].

2 result See Table 1 for the reliability and validity indicators of the Athlete Training State Monitoring Scale (32 × 7). The composite reliability (CR) and average variance extracted (AVE) estimated by MLE and Bayesian methods were comparable. Most of the CR indicators are greater than 0. 70, and most of the AVE indicators are greater than 0. 36, suggesting good internal reliability and convergent validity [1]. The discriminant validity estimated by the Bayesian method is good, and the Pearson correlation between all factors is smaller than the square root of the average variance extraction amount[26].

Table 1 Internal reliability, convergent validity, discriminant validity

dimension	internal reliability (CR)	convergent validity (AVE)	discriminant validity 1	2	3	4	5	6	7	8
emotional stress	0. 752 (0. 697)	0. 433 (0. 366)	(0.605) 0. 658	-0.170	0. 280	-0. 172	-0. 172	-0. 175	0. 285	0. 288
feel good	0. 804 (0. 730)	0. 513 (0. 406)	-0.349	(0.637) 0. 716	-0. 166	0. 269	0. 269	0. 274	-0.170	-0. 172
tired labor	0. 695 (0. 694)	0. 370 (0. 364)	0. 795	-0.321	(0.603) 0. 608	-0. 168	-0. 168	-0.171	0. 279	0. 282
self efficacy	0. 751 (0. 732)	0. 431 (0. 406)	-0.403	0. 548	-0.371	(0.637) 0. 657	0. 272	0. 277	-0. 172	-0. 174
self-regulation	0. 773 (0. 745)	0. 464 (0. 422)	-0.386	0. 524	-0.355	0. 605	(0. 650) 0. 681	0. 277	-0. 172	-0. 174
physical recovery	0. 747 (0. 693)	0. 429 (0. 361)	-0.480	0. 652	-0.442	0. 753	0. 720	(0.601) 0. 655	-0. 175	-0. 177
mental exhaustion	0. 799 (0. 781)	0. 501 (0. 472)	0. 625	-0. 253	0. 576	-0. 292	-0. 279	-0.347	(0. 687) 0. 708	0. 288

mental fatigue	0. 755 (0. 746)	0. 440 (0. 424)	0. 638	-0. 258	0. 587	-0. 298	-0. 285	-0.354	0. 461	(0. 651) 0. 663

Note: The values in brackets are the estimation results of Bayesian method, and the values outside the brackets are the estimation results of maximum likelihood method. Discriminant validity The upper right corner is the latent variable Pearson correlation coefficient estimated by the Bayesian method, the lower left corner is the latent variable Pearson correlation coefficient estimated by the maximum likelihood method, and the diagonal line is the square root of the average variance extraction.

Part of the fitting index is not ideal (P < 0. 05, 0. 05 > RMSEA and SRMR > 0. 08, TLI and CFI < 0. 90). In addition, the residual variance of the two factors of emotional stress and physical recovery is not significant, which means that the estimated value of this parameter is very close to a negative value, which is beyond the acceptable range of statistical analysis. The model correction index suggested that 5 items may have cross loading (the correction index is 12 ~ 23), and 10 items may have residual correlation (the correction index is 12 ~ 25). Maximum likelihood estimation does not allow cross loadings and residual correlations; therefore, it is common practice not to add cross loadings and Instead, try to delete the entry with the largest chi-square change according to the prompt of the correction index.

The fitting indexes of Bayesian estimation model under three different prior information conditions are shown in Table 2. All model PSR indicators are less than 1.05 after 50,000 iterations. Checking all the parameter locus diagrams also indicated that they had converged.

For the uninformative prior model, including factor loading and cross loading prior information model, the PPp value is close to 0, and the lower limit of 95% CI is positive, indicating that the model is poorly fitted. However, the PPp of the model incorporating factor loads, cross loads, and residual related prior information was 0. 572, and it had a 0-centered, basically symmetrical 95% CI, and the model fitted well. In addition, the DIC index of this model is smaller than that of the uninformative prior model, which is not much different from the other model. Overall, the models that incorporated prior information on factor loadings, cross loadings, and residuals fit better.

The measurement model factor loads and cross loads are shown in Table 2. The factor loadings estimated by MLE were 0. 516 to 0. 868 except for one item which was slightly lower (0. 451). All factor loadings were significant at the 0. 001 level. The factor loadings estimated by the Bayesian method ranged from 0. 521 to 0. 721, and the 95% CIs of all factor loadings did not contain 0. Almost all crossloads

The charges were all within ± 0. 28, and the 95% CI contained 0. The factor loading and cross loading estimated by Bayesian method are within the acceptable range.

Table 2 Factor loading, Cross Loading and Path Coefficient

Question number	emotional stress	feel good	fatigue	self efficacy	self-regulation	physical recovery	mental exhaustion	mental fatigue
Q01	0. 595* 541)	(0.-0.131	0. 105	-0.029	-0.006	0.006	0. 019	-0.023
Q09	0. 636* 679)	(0.-0.099	0. 126	-0.039	-0.004	0.041	0.061	0. 058
Q17	0. 597* 712)	(0.-0. 196*	0. 121	0. 018	0. 013	-0.014	-0.015	0. 116
Q25	0. 590* 687)	(0.-0.088	0. 181*	0. 069	0.057	-0.051	0.091	0. 014
Q02	-0. 163	0. 638* 619)	(0.0.026	0. 034	-0.023	0.043	-0.001	-0.030
Q10	-0.116	0. 676* 755)	(0.0.007	0. 146	-0.018	0. 012	-0.070	0.009
Q18	-0.158	0. 699* 868)	(0.-0.058	0.066	0.034	0. 068	0.037	-0.033
Q26	-0.111	0. 521* 587)	(0.-0. 148	0. 130	0.070	0.041	0.001	-0.014
Q03	0. 108	-0.068	0. 648* 569)	(0.-0.021	0.026	-0.032	-0.040	-0.044
Q11	0.054	0.030	0. 530* 451)	(0.0. 059	-0.073	-0. 143	0.020	-0.030
Q19	0.061	0.067	0. 678* 619)	(0.0.061	0. 017	-0.008	0. 018	0. 122
Q27	0. 315*	-0.077	0. 544* 756)	(0.0.080	0.099	-0.037	0.063	0. 019
Q04	0.074	0. 059	-0.002	0. 630* 649)	(0.0. 114	0. 111	0.094	0.020
Q12	-0.015	0.091	0.026	0. 677* 729)	(0.-0.018	0. 161	0.091	-0.022
Q20	0. 034	0. 139	0. 132	0. 641* 616)	(0.0. 058	0. 069	0. 023	-0.002
Q28	-0.007	0.070	0.010	0. 599* 626)	(0.0. 151	0.067	-0.030	0.031

stress	0. 534* (0. 929)		0. 523* (0. 856)				0. 534* (0. 672)	0. 539* (0. 686)
recover		0. 516* (0. 689)		0. 522* (0. 795)	0. 522* (0. 761)	0. 531* (0. 947)		

Note: The values in brackets are the results of maximum likelihood estimation, and all estimated values are significant at the 0.001 level; the values outside brackets are the results of Bayesian estimation. * Indicates that the 95% CI does not contain 0.

Finally, of the 496 residual correlations that measure the model, the 95% CI does not contain 0 for only 4. The mean of the absolute value of all residual correlations is 0. 063, and the range of residual correlations is -0. 284 ~ 0. 278. This result suggests that a large number of small residual correlations need to be modeled, as expected.

twenty three analyze The CFA model, which incorporates prior information about cross loadings and residuals, fits well, while the results of maximum likelihood estimation show a poor model fit. The reason may be that the maximum likelihood estimation does not include cross loading and residual correlation into the analysis process [6]. It is well known that assessment instruments such as standardized scales often contain small but non-zero cross loadings and residual correlations. The maximum likelihood estimation fixes the cross loading and residual correlation to 0, just for the sake of model simplicity, not to make the model more realistic. In order to improve the fitting degree of the model, researchers often take the risk of opportunity expansion and make post-event corrections to the model according to the correction index [5]. This approach not only has the risk of missing potential and meaningful theoretical structures [27], it is more likely to put researchers in a dilemma. When the model is corrected after the fact based on the correction index, the primary consideration is whether the theoretical basis is sufficient and whether the revised model is interpretable [28] ; But once a sufficient reason for revising the model is given, one will immediately face the following question: Since the reason is so sufficient, why is it not reflected in the original model? [5] CFA using Bayesian estimation allows the crossover The load and residual correlation are included in the model for analysis, so as to effectively avoid the above problems [6]. In addition, the method bias caused by the similarity of the scale topic structure, content and wording, and the consistency of the scale reflection form will be included in the residuals of each measurement item. The residual correlation caused by the common method bias can be reflected in the model, which is also a possible explanation for the good fit of the Bayesian estimation model incorporating the residual correlation.

The structural equation model requires the error variance to be positive and significant [1]. In the case where the error variance is not significant in the maximum likelihood estimation result, it is an inappropriate solution. This is one of the problems often encountered in maximum likelihood estimation [29-30]. This situation in the case study results is likely due to the small sample size. According to the rule of thumb for determining the sample size by HAIR et al., when the model contains more than 7 constructs, the average variance extraction of some constructs is low (such as less than 0.45), and (or) the observed items of a single construct are less than 3 The minimum sample size of research cases should be 500. The error variances estimated by the Bayesian method were all positive, and none of the 95% CIs contained 0, suggesting that there was no inappropriate solution. With the help of Gibbs sampling algorithm and the inclusion of prior information of residual variance and covariance, the Bayesian method can effectively avoid the situation of inappropriate solutions [6-7]. In the practice of sports and exercise psychology research, due to the difficulty of sampling (such as collecting data from Olympic athletes), sample loss (such as longitudinal follow-up research), and strict screening criteria (such as examining the adjustment effect), researchers often have to Faced with small sample data [31]. The Bayesian method can help researchers to effectively deal with the difficulty (or even impossibility) of obtaining large samples due to various conditions by means of the Gibbs sampling algorithm and the inclusion of prior information [6].

There was a large difference in latent variable correlations estimated by MLE and Bayesian methods (Table 1). The results of maximum likelihood estimation showed that the Pearson correlation coefficients among some factors were greater than 0. 75, suggesting that there may be multicollinearity problems. Multicollinearity problems can lead to uninterpretable results, commonly used

The solution is to remove some independent variables, which is not conducive to the effective testing of the hypothesis model [28]. And Bayesian estimation can solve the problem of multicollinearity. After incorporating the small variance prior information related to the cross load and the residual, the correlation between the items is no longer reflected through the latent variable correlation [6], which can reduce the latent variable correlation coefficient. On the contrary, CFA using maximum likelihood estimation does not consider cross loading and residual correlation, and latent variable correlation is often overestimated [3], which is also the reason for the better discriminant validity of Bayesian estimation.

3. DISCUSS ARGUMENT

This study briefly introduces BSEM and illustrates the analysis process of BSEM through a real second-order CFA. Compared with the maximum likelihood estimation, the BSEM analysis process has greater flexibility, and can more properly reflect the researcher's theoretical conception and prior belief [6]. From the analysis process of the research case, we can see that: 1) Bayesian method allows to estimate the cross loading and residual correlation in the model, so as to effectively avoid poor model fitting, biased estimation of factor correlation, and expansion of opportunities caused by model revision, etc. question. 2) Under the same conditions, Bayesian estimation requires less sample size than maximum likelihood estimation. According to the research of LEE et al. [32], BSEM only needs 2 to 3 times the sample size of the free parameters, while the maximum likelihood estimation requires 4 to 5 times the sample size of the free parameters. Generally speaking, under the condition of large sample size and all parameters obey the normal distribution, the results of maximum likelihood estimation and Bayesian estimation have little difference. However, Bayesian estimation can still obtain more accurate parameter estimates even under the condition of small sample size. 3) By setting the prior distribution of residual covariance with small variance, BSEM can effectively avoid estimated values beyond the acceptable range of statistical analysis such as "residual variance is negative" and "standardized regression coefficient is greater than 1".

Of course, BSEM also has voices of doubt. One of the most common objections is that the choice of prior is mainly based on the subjective decision of the researcher [33] ; Therefore, researchers need to clearly and clearly describe the source and basis of prior information to ensure the transparency of prior selection, and it is necessary to test the influence of prior selection on the posterior distribution by setting different prior information [9]. Stromeyer et al. [34] questioned several technical issues of BSEM, such as the lack of guidance on the use of residual covariance, cross-loading may mask problematic measurement models, and so on. ASPAROU-HOV et al. [35] conducted an in-depth investigation on the above issues through the analysis of simulated data and real data, and reanalyzed the sample data used by STROMEYER et al. The results supported the effectiveness of BSEM.

Although Bayesian methods, including BSEM, still have many issues to be discussed [34-37], it is undeniable that Bayesian methods are increasingly used in psychology, including the field of sports and exercise psychology. VAN DE SCHOOT et al. [38] reported after examining the papers published in the field of psychology from 1990 to 2015 that among the psychology papers published each year, the proportion of papers mentioning Bayesian methods increased from 10% in 1990 to increased to 40% in 2015. In the field of sports and exercise psychology, there have been many articles published in the field of BSEM in recent years [25, 31, 39-42]. Compared with classical statistical methods, Bayesian methods provide researchers with a lot of convenience in theory and practice [8]. The Bayesian method obtains updated knowledge by integrating previous knowledge and experience with current observation data, which embodies the inheritance of knowledge accumulation. With the popularization and application of Bayesian methods in the domestic field of sports and exercise psychology, it will promote the transformation of knowledge accumulation methods in this field.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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