

DECISION OPTIMIZATION IN THE PRODUCTION PROCESS BASED ON DYNAMIC PROGRAMMING

ZeBin Wang¹, SaiNan He¹, YuDong Zhao¹, ZhengBo Li^{2*}

¹Department of Automation for Brewing Engineering, MouTai Institute, Ren Huai 564500, GuiZhou, China.

²Department of Public Basic Education, MouTai Institute, Ren Huai 564500, GuiZhou, China.

Corresponding Author: ZhengBo Li, Email: wxxtedu@sina.cn

Abstract: This study proposes an optimization method for production decisions in the manufacturing process of popular electronic products, focusing on issues such as component procurement, product assembly, and quality inspection. In the production process, companies need to purchase key components, and any defective part may lead to a substandard final product. For defective products, the company may choose to either scrap or disassemble them for recycling. This paper employs a binomial distribution model to design a sampling inspection scheme, establishes a mathematical model, and determines the sample size at different confidence levels to accurately assess whether the defect rate exceeds the threshold. Additionally, this paper integrates various factors to construct a production cost minimization model, identifying the optimal production strategy in most scenarios. This strategy includes comprehensive inspections of both components and finished products, as well as disassembly and recycling plans for defective products. The results show that optimizing production decisions can significantly improve production efficiency, reduce costs, and enhance the company's market competitiveness.

Keywords: Binomial distribution model; Production cost minimization; Multi-stage decision optimization; Production process efficiency

1 INTRODUCTION

With the increasing popularity of electronic products and the growing market demand, efficiently and economically producing high-quality products has become a major challenge for companies. In recent years, many scholars have conducted relevant research in this field, proposing various optimization methods and models. ShaoBo studied a product sampling model based on cost and quality control, proposing a strategy to reduce product quality inspection costs through controlled sampling inspection [1]; ZhangBianya calculated the average number of products in a queue and the average waiting time, analyzed the key factors influencing production efficiency, and proposed an optimization scheme to shorten processing time by adjusting the number of workshop processors [2]; ZhangMengyan proposed a fuzzy multi-objective nonlinear optimization algorithm based on opportunity constraints, using membership functions and Taylor expansion methods for optimization [3].

Although the aforementioned studies have made significant progress in optimizing the product production process, most of the research focuses on optimizing single production stages and rarely addresses the systemic and dynamic nature of the entire production process. Moreover, existing studies often fail to adequately consider the impact of market changes and personalized demands on production decisions, factors that play a critical role in actual production.

To address the gaps in current research, this paper proposes a more comprehensive and dynamic production process optimization method based on existing studies. This method not only covers all production stages from raw material procurement to finished product assembly, but also introduces a dynamic programming model, allowing production decisions to be adjusted in real time according to market demand and production conditions. Furthermore, by incorporating big data analysis technology, this paper proposes a decision support system to enhance the scientific accuracy of production decisions, thereby enabling enterprises to formulate more rational production strategies, reduce production costs, improve production efficiency, and enhance market competitiveness.

2 MODEL ESTABLISHMENT AND SOLUTION OF THE SAMPLING INSPECTION SCHEME

2.1 Establishment of the Binomial Distribution Model

In this study, we aim to determine whether a batch of components should be accepted or rejected based on a small number of inspections. Among several models, including the Poisson distribution model, Bayesian probability model, conditional probability model, and binomial distribution model [4], the binomial distribution model is chosen for analysis. Its advantage lies in its ability to accurately describe the probability distribution of the number of successes in a fixed number of independent Bernoulli trials [5].

2.1.1 Hypothesis verification

Using the binomial distribution for modeling, the status of each component in the sample inspection can be seen as a result of a binomial distribution. Let the probability of rejection be denoted as p_0 , then $p_0 > 10\%$ for rejection, and $p_0 < 10\%$ for acceptance. The probability mass function for a binomial distribution is given by:

$$p(x = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{1}$$

Where X is the number of defective products in the sample, n is the sample size, p is the defect rate, and k is the number of defective products observed.

2.1.2 Approximation of the binomial distribution to the normal distribution

The normal distribution model is considered an advanced conceptual model that clearly and intuitively displays dynamic changes in data [6]. Due to the complexity of the binomial distribution, when the sample size is large, it can be approximated by the normal distribution:

$$x \sim n(np, np(1 - np)) \tag{2}$$

Based on statistical tests, the test statistic for the binomial distribution can be simplified as [7]:

$$z = \frac{p_0 - p}{\sqrt{\frac{p_0(1 - p_0)}{n}}} \tag{3}$$

2.2 Solution of the Model

At a 95% confidence level, the rejection region corresponds to a Z value of 1.645. Therefore, the condition for rejecting the batch of components is $Z > 1.645$. At a 90% confidence level, the acceptance region corresponds to a Z value of 1.282. Thus, the condition for accepting the batch is $Z < 1.282$. The sample size can be determined by:

$$n = \frac{(p_0 - p)^2}{z^2 p(1 - p_0)} \tag{4}$$

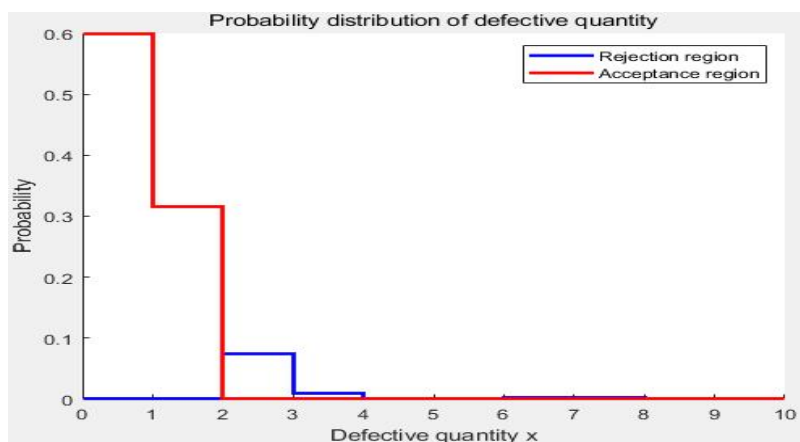


Figure 1 Probability Distribution of the Number of Defective Products

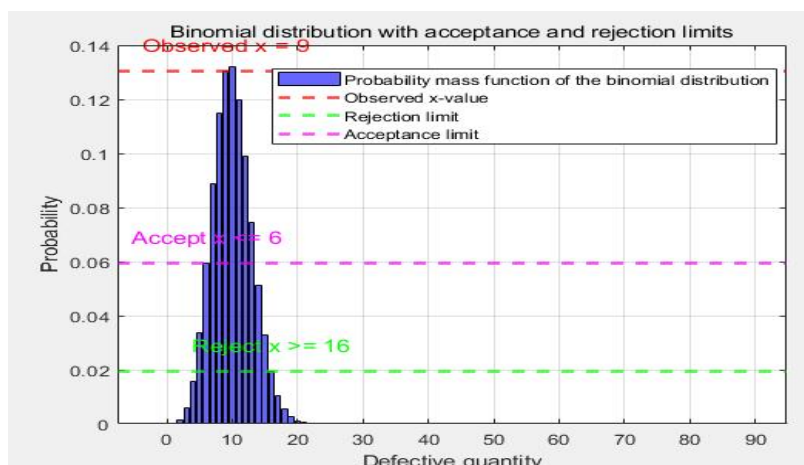


Figure 2 Binomial Distribution with Acceptance and Rejection Limits

From Figures 1 and 2, it can be observed that at a 95% confidence level, in order to protect product quality and the company's reputation from the potential damage of a high defect rate, the sample size is set to 98. In this case, if the number of defective items exceeds the preset critical value, the batch of components will be rejected. This decision standard aims to effectively prevent potential quality risks.

At a 90% confidence level, in order to balance quality control with cost control, the sample size is set to 60. Under this

standard, if the number of defective items does not exceed (i.e., is less than or equal to) the critical value, the batch of components will be accepted as meeting the quality standard.

3 MODEL ESTABLISHMENT AND SOLUTION OF PRODUCTION PROCESS DECISIONS

This study focuses on the following four decision areas that the company needs to make in the production process:

Component Inspection Decision: Whether to inspect the components to prevent defective parts from entering the assembly stage.

Finished Product Inspection Decision: Whether to inspect the assembled finished products to ensure their quality before they enter the market.

Disassembly of Defective Products: Whether to disassemble defective products for recycling or to scrap them.

Customer Exchange Decision: Whether to provide unconditional exchanges for defective products purchased by customers and bear the associated costs.

3.1 Establishment of the Decision Model

3.1.1 Decision variables

d_1 、 d_2 : Binary decision variables indicating whether to inspect component 1 and component 2 respectively ($d_i=1$ indicates inspection, $d_i=0$ indicates no inspection).

d_f : Binary decision variable indicating whether to inspect the assembled finished product ($d_f=1$ indicates inspection, $d_f=0$ indicates no inspection).

d_r : Binary decision variable indicating whether to disassemble a defective finished product ($d_r=1$ indicates disassembly, $d_r=0$ indicates no disassembly).

3.1.2 Decision logic and cost calculation

(1) Component Inspection Decision

In manufacturing, the decision to inspect components before assembly is crucial. If the components are inspected, although some inspection costs are incurred, the risk of defective components leading to scrapped final products is effectively avoided. On the other hand, if the inspection is skipped, the risk of having quality issues in the assembled product remains. Furthermore, the decision of whether to inspect the assembled finished products before they are released to the market is equally important. If finished products are not inspected, defective products entering the market can result in compensation costs far exceeding the inspection cost. By conducting a final product inspection, the quality of products entering the market can be assured.

For defective products identified during inspections, a further decision must be made regarding whether to disassemble them. If disassembly is chosen, additional disassembly costs are incurred, but it helps in resource recovery and reuse. If disassembly is not chosen, the products will be scrapped, resulting in resource waste [8].

(2) Cost Calculation

The procurement quantity for Component 1 is calculated as:

$$\frac{n}{(1-r_1(1-d_1))} \quad (5)$$

The procurement cost for Component 1 is:

$$\frac{n}{(1-r_1(1-d_1))} c_{p1} \quad (6)$$

The inspection cost for Component 1 is:

$$d_1 \frac{n}{(1-r_1)} c_{i1} \quad (7)$$

Similarly, the procurement and inspection costs for Component 2 are calculated as:

$$\frac{n}{(1-r_2(1-d_2))} \quad (8)$$

$$\frac{n}{(1-r_2(1-d_2))} c_{p2} \quad (9)$$

$$d_2 \frac{n}{(1-r_2)} c_{i2} \quad (10)$$

The assembly cost for the finished product is:

$$nc_a \quad (11)$$

The inspection cost for the finished product is:

$$d_f nc_{if} \quad (12)$$

If no inspection is performed, defective products enter the market, causing replacement losses:

$$(1 - d_f)nr_f c_e \tag{13}$$

The disassembly cost for defective products is:

$$d_r n(1 - p_{\text{Qualified}})c_r \tag{14}$$

$$p_{\text{Qualified}} = (1 - r_1(1 - d_1))(1 - r_2(1 - d_2))(1 - r_f(1 - d_f)) \tag{15}$$

Where n is the total sample size, r_1 is the defect rate of component 1, r_2 is the defect rate of component 2, c_r is the purchase unit price of component 1, and c_e is the purchase unit price of component 2.

3.2 Solution of the Model

Through the developed model, we can solve for the optimal production process decision by evaluating different decision combinations and their associated costs, as shown in Figure 3. These combinations cover a variety of possible strategies for inspecting and disassembling products. The cost analysis provides a deeper understanding of which strategies are most effective in reducing total costs while ensuring product quality [9].

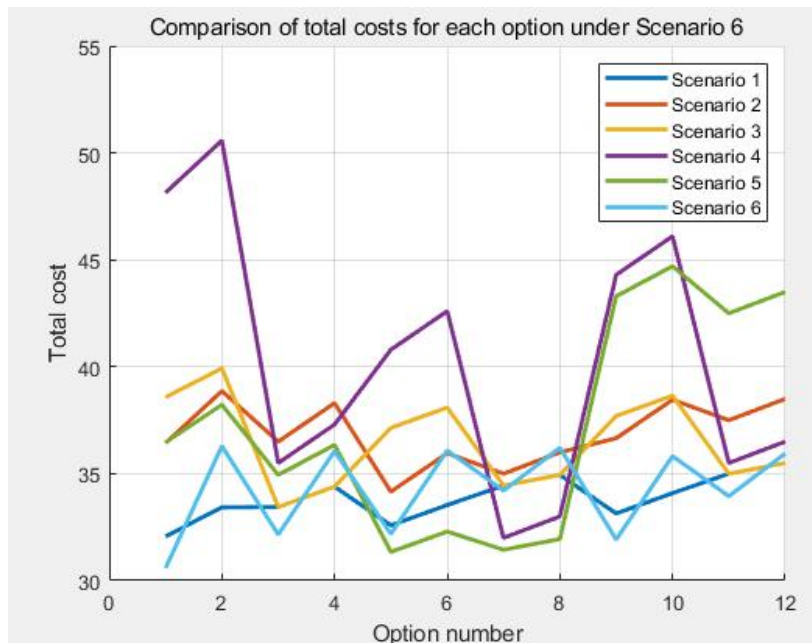


Figure 3 Comparison of Total Costs for Each Scenario under 6 Situations

3.3 Analysis of Production Process Decision Results

Through the above scenario cost charts, six situations can be obtained as shown in Table 1, and the optimal total cost is shown in Figure 4. (Use 1 to indicate detection, and 0 to indicate no detection).The comparison of the decision alternatives under different scenarios helps the company make informed decisions regarding cost optimization and resource allocation.

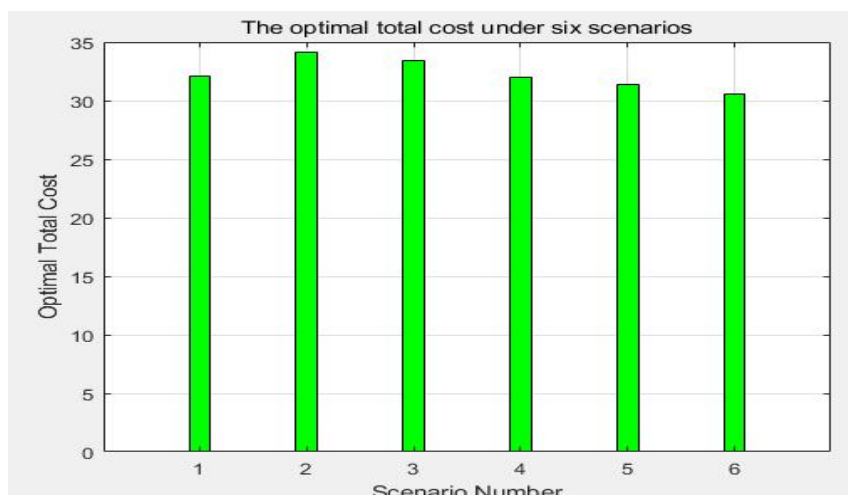


Figure 4 Optimal Total Cost under Six Scenarios**Table 1** Inspection Table for Each Scenario

Scenario	Optimal solution	Component 1	Component 2	Finished product	Disassembly of non-conforming finished products
1	1	0	0	0	0
2	5	0	1	0	0
3	3	0	0	1	0
4	7	0	1	1	0
5	5	0	1	0	0
6	1	0	0	0	0

4 MODEL ESTABLISHMENT AND SOLUTION FOR MULTI-STAGE DECISION OPTIMIZATION

This section investigates multi-stage decision optimization problems, involving the inspection and processing decisions for each process step. The ultimate goal is to find an optimal set of strategies that minimize the overall cost throughout the production process. The costs to be optimized include: deciding whether to inspect components; deciding whether to inspect the finished products; whether to dismantle or scrap non-conforming products, or conduct replacements after they have entered the market; the potential impact of defective products during assembly on the final product quality; and the additional replacement costs incurred by the company if defective products reach the market. In such problems, we typically aim to reduce the overall production costs by controlling the frequency of inspections and processing strategies, while ensuring quality [10].

4.1 Model Establishment

4.1.1 Cost of component inspection

In this paper, it is assumed that there are n types of components, and in m processes, whether to inspect the components in each process can be chosen. The procurement quantity of parts:

$$m_{\text{Quantity of Part N}} = \frac{y}{(1-r_x(1-d_x))} \quad (16)$$

Procurement cost and inspection cost for (n) types of parts:

$$m_{\text{Procurement Cost of Part N}} = \sum_{x=1}^n \frac{y}{(1-r_x(1-d_x))} c_{px} \quad (17)$$

$$m_{\text{Inspection Cost of Part N}} = \sum_{x=1}^n d_x \frac{y}{(1-r_x)} c_{tx} \quad (18)$$

d_x is the inspection decision for component n , r_x is the defect rate of component n , c_{px} is the purchase price per unit of component n , and c_{tx} is the inspection price per unit of component n .

4.1.2 Semi-finished product inspection cost

In this paper, it is assumed that there are y types of semi-finished products, and whether to inspect the y types of semi-finished products. The assembly cost and inspection cost for semi-finished product y composed of n types of parts:

$$m_{\text{Assembly Cost of Semi-finished Products}} = \sum_{y=1}^y y c_{ay} \quad (19)$$

$$m_{\text{Inspection Cost of Semi-finished Products}} = \sum_{y=1}^y d_{fy} y c_{ty} \quad (20)$$

c_{ay} is the assembly price per unit of the semi-finished product, c_{ty} is the inspection price per unit of the semi-finished product, and d_{fy} is the inspection decision for semi-finished product (y).

4.1.3 Finished product inspection cost

The finished product inspection determines whether the finished product enters the market. If inspected, it can prevent defective products from entering the market. The assembly cost and inspection cost of the finished product:

$$m_{\text{Assembly Cost of Finished Products}} = \sum_{y=1}^y y(1-r_{fy}) r_f c_a \quad (21)$$

$$m_{\text{Inspection Cost of Finished Products}} = \sum_{y=1}^y y(1-r_{fy}) r_f c_{if} \quad (22)$$

r_{fy} is the defect rate of the semi-finished product, r_f is the defect rate of the finished product, c_a is the assembly price per unit of the finished product, and c_{tf} is the inspection price per unit of the finished product.

4.1.4 Handling of non-conforming products

In the production process, non-conforming products can be detected through inspection and handled, with two handling options: disassembly and recycling, or direct scrapping. Semi-finished product disassembly cost:

$$m_{\text{Disassembly Cost}} = \sum_{y=1} d_{ry} y (1 - p_{\text{Qualified}}) c_{ry} \tag{23}$$

$$p_{\text{Qualified}} = \prod_{x=1} \prod_{y=1} ((1 - r_x (1 - d_x)) (1 - r_{fy} (1 - d_{fy}))) \tag{24}$$

Cost of finished product disassembly:

$$m_{\text{Disassembly Cost}} = \sum_{y=1} d_r y (1 - r_{fy}) r_f (1 - p_{\text{Qualified}}) c_r \tag{25}$$

$$p_{\text{Qualified}} = (\prod_{x=1} \prod_{y=1} (1 - r_x (1 - d_x)) (1 - r_{fy} (1 - d_{fy}))) (1 - r_f (1 - d_f)) \tag{26}$$

Total cost of disassembly:

$$m_{\text{Total Cost}} = \sum_{y=1} d_{ry} y (1 - p_{\text{Qualified}}) c_{ry} + \sum_{y=1} d_r y (1 - r_{fy}) r_f (1 - p_{\text{Qualified}}) c_r \tag{27}$$

4.1.5 Assembly and replacement loss

When certain components are not inspected or defective finished products enter the market, the company may incur market replacement losses. The cost of replacement due to non-inspection entering the market:

$$m_{\text{Finished Product Replacement Cost}} = \sum_{x=1} \sum_{y=1} y (1 - r_{fy}) r_f c_e \tag{28}$$

c_e is the unit price of replacement loss.

4.1.6 Total cost of assembling into finished products

$$m_{\text{Total Cost}} = \sum_{x=1} \frac{y}{(1 - r_x (1 - d_x))} c_{px} + \sum_{x=1} d_x \frac{y}{(1 - r_x)} c_{tx} + \sum_{y=1} d_{fy} y c_{ty} + \sum_{y=1} y (1 - r_{fy}) r_f c_a \tag{29}$$

$$+ \sum_{y=1} d_{ry} y (1 - p_{\text{Qualified}}) c_{ry} + \sum_{y=1} d_r y (1 - r_{fy}) r_f (1 - p_{\text{Qualified}}) c_r + \sum_{x=1} \sum_{y=1} y (1 - r_{fy}) r_f c_e$$

4.2 Model Solution

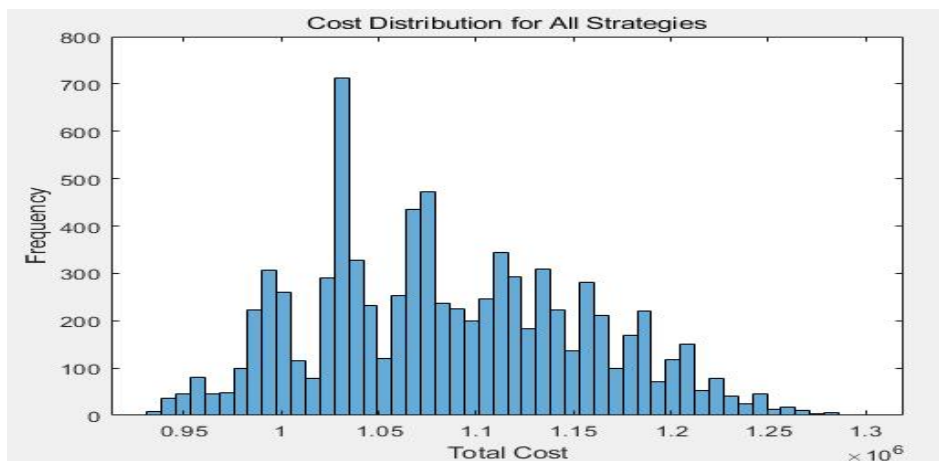


Figure 5 Partial Cost Breakdown Chart

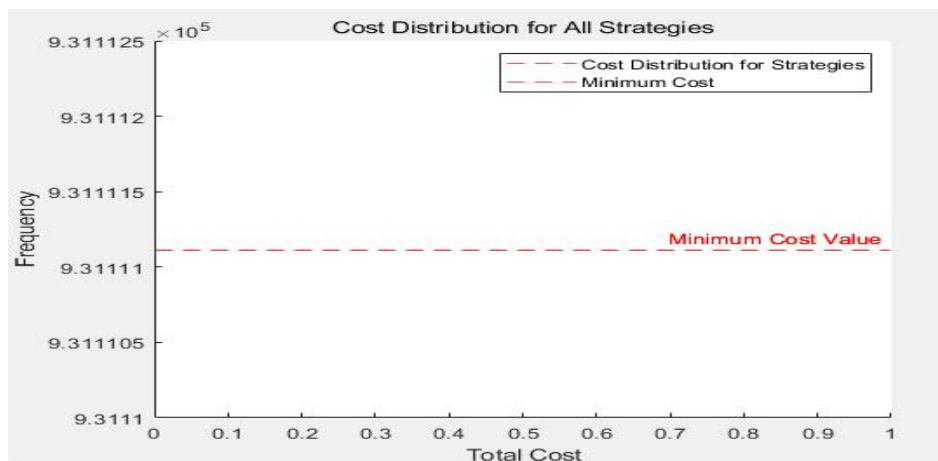


Figure 6 Minimum Cost Value

As can be seen from Figures 5 and 6: By analyzing the costs of different decision combinations, the company can choose a strategy of comprehensive non-inspection and disassembly of non-conforming finished products to effectively control the defect rate and reduce losses. Although this decision incurs inspection costs and disassembly fees, it can minimize the total cost in the long-term production.

5 CONCLUSION

This model has significant application value in manufacturing quality control and cost optimization. Through calculation and graphical analysis, this paper concludes that at a 95% confidence level, to protect product quality, the sample size is set to 98, and strict rejection criteria are established; at a 90% confidence level, to control costs, the sample size is adjusted to 60, achieving a balance between quality and cost. Comprehensive inspection of components and finished products, especially when the defect rate is high or the inspection cost is low, can reduce economic losses. For non-conforming finished products, appropriate measures are taken according to the disassembly cost to promote resource reuse. These strategies reflect the flexibility and specificity of the model, supporting enterprises to improve product quality, effectively manage costs, and enhance competitiveness. The advantage of the model lies in simplifying the production process, quality control, and cost assessment, making it widely applicable.

In the future, this research can be expanded in multiple ways. Firstly, artificial intelligence technologies can be introduced to mine production data through machine learning algorithms, thus accurately predicting the defective rate. For instance, deep - learning models can be used to analyze data such as equipment parameters and raw material characteristics, detect quality risks in advance, optimize the inspection strategy, and achieve intelligent quality control. Secondly, in combination with supply chain management, factors such as supplier stability, delivery time, and fluctuations in raw material prices can be incorporated into the model to construct a more complete production decision - making system, enhancing enterprises' ability to respond to supply - chain risks. Thirdly, explore the differences in the application of the model in different sub - fields of the manufacturing industry. For industries with extremely high requirements for quality and cost control, such as automotive and electronic chip manufacturing, refine the model parameters, improve adaptability and accuracy, provide customized decision - making optimization solutions for enterprises, and promote the high - quality and high - efficiency development of the manufacturing industry.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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