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SPATIOTEMPORAL VARIABILITY OF YELLOW RIVER WATER-SEDIMENT FLUXES: A HYBRID APPROACH USING CUBIC SPLINE INTERPOLATION AND MANN-KENDALL TEST

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Abstract: Studying the variation law of water and sediment fluxes in the Yellow River is of great significance for environmental protection, adaptation to climate change and improvement of the quality of life of residents in the Yellow River Basin. this paper established a cubic spline interpolation model to supplement the missing sediment content data. By using the known data and interpolation data for plotting, it was found that the interpolation effect was good. Subsequently, the total drainage volume and total sediment discharge volume from 2016 to 2021 were calculated respectively using the interpolation data. In order to further study the variability of water and sediment content, this paper adopts the Mann-Kendall non-parametric test method to analyze the variability of water and sediment flux, revealing its inherent law. Through specific data analysis, the important role of sand regulation and water control in ensuring the ecological health of the Yellow River has been further demonstrated.
Keywords: Cubic spline interpolation; Mann-Kendall non-parametric test method; Residual analysis; MATLAB

1 INTRODUCTION

The Yellow River plays a pivotal role in shaping the ecological and socioeconomic fabric of its basin. Investigating the spatiotemporal variability of water-sediment flux in the Yellow River holds critical theoretical significance for addressing environmental governance, climate change adaptation, and livelihood improvements across the basin. Furthermore, it provides essential insights into optimizing water resource allocation, coordinating human-land relationships, regulating water-sediment discharge, and enhancing flood control and disaster mitigation strategies. As one of China's most vital river systems, the Yellow River's hydrological and sediment dynamics exert profound impacts on ecosystem integrity, flood resilience, and sustainable socioeconomic development. Characterized by exceptionally high sediment concentrations, the Yellow River presents unparalleled challenges in water-sediment management, necessitating highly accurate, real-time, and granular monitoring systems. Advanced analysis of water-sediment dynamics enables the identification of temporal patterns, prediction of future flux trends, and formulation of science-driven policies for ecological conservation, flood risk reduction, and adaptive river basin management.

Firstly, the complexity of the Yellow River water-sediment regulation system and its impact on basin sustainable development have become focal points of research. Cao et al. analyzed the influencing factors and evolution trends of the Yellow River water-sediment regulation system from a systemic perspective, identifying climate change and human activities as the key drivers of abrupt changes in water-sediment relationships [1]. Current models still have limitations in their response mechanisms to extreme hydrological events, particularly in terms of prediction accuracy under high sediment load scenarios. Secondly, addressing the issue of missing monitoring data, cubic spline interpolation has emerged as a mainstream method due to its smoothness and local adaptability. Habermann and Kindermann systematically expounded the mathematical principles of multidimensional spline interpolation [2], demonstrating its superiority in reconstructing non-uniform data; Abdulmohsin et al proposed a classification method based on cubic spline interpolation, verifying its robustness in high-noise environments [3]; He and Li combined it with support vector quantile regression for uncertainty analysis in wind power probability density prediction, achieving a 15% reduction in error [4]. Liu et al. proposed a Transformer-based model with missing position encoding, leveraging self-attention mechanisms to capture long-term dependencies in multivariate hydrological sequences, achieving a 15%–25% reduction in imputation errors [5]. In Yellow River water-sediment studies, cubic spline interpolation has been applied to complete sediment concentration data, but its accuracy in edge regions is constrained by data sparsity. Future research could explore adaptive weighted interpolation or multi-scale fusion algorithms to enhance reliability. Thirdly, non-parametric statistical methods hold significant importance in hydrological trend analysis, with the Mann-Kendall (MK) test being the most widely applied. Mann and Kendall established the theoretical foundation of the MK test [6], whose characteristic of not requiring assumptions about data distribution makes it suitable for non-normal hydrological sequences; Patle et al. and Cabral Júnior et al. respectively applied the MK test to precipitation trend analysis in India and Brazil, finding it maintains high sensitivity even with small sample sizes [7]; Sang et al. compared the MK test with empirical mode decomposition (EMD), noting the MK test's greater advantage in detecting abrupt changes but its limitation in distinguishing natural fluctuations from human interventions [8].

Through reading literature, it has been found that there is limited research on the use of cubic spline interpolation in the study of water and sediment fluxes in the Yellow River. Therefore, this article proposes using cubic spline interpolation to study the water and sediment flux of the Yellow River.

2 MODEL

2.1 Cubic Spline Interpolation

The cubic spline interpolation is a method that uses a series of cubic polynomials assembled according to certain smoothness requirements to approximate a function or a set of data points.
Divide the data range into multiple sub-ranges and define a cubic polynomial for each sub-range.

$$S_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i \tag{1}$$

where $i = 1, 2, 3 ... n-1$; where a_i, b_i, c_i, d_i are the coefficients of cubic spline interpolation.
Constrains adjacent polynomials to be continuous in function values and first and second derivatives at the junction points.
Combined with boundary conditions (such as natural splines) to solve the coefficient, the global smooth curve is formed.
The data utilized in this article is sourced from the website www.mcm.edu.cn..
The relationship between sediment concentration and time, water level and discharge in a hydrology station from 2016 to 2021 was studied. Supplement the missing annual sediment concentration data according to the known data.
The steps to supplement the sediment content data are as follows (take 2016 as an example) :
Step 1: Data selection
Looking at six years of monitoring data, it was found that when filtering the data with the keyword "sediment concentration", there were 373 sampling events in 2016, of which 366 occurred at 8 am, accounting for a certain proportion. The statistical results of other years are shown in Table 1.

Table 1 Statistical Table of Sediment Concentration at 8:00 from 2016 to 2021

Year	Total sampling times of sediment content	8:00 Sampling times of sediment content	ratio
2016	373	366	98.1%
2017	372	358	96.2%
2018	432	364	84.2%
2019	404	364	90%
2020	321	274	85.3%
2021	257	227	88.3%

As can be seen from Table 1, in the past six years, the data of sediment content sampled by the hydrological station at 8:00 am every day accounted for 85% of the total data. Therefore, in order to fill in the missing sediment concentration data, the annual 8 a.m. sampling data was used.
Step 2: Time processing
Insert a serial number column in the 2016 data , each serial number corresponding to a specific date and time. For example, serial number 1 corresponds to 0:00 a.m. on January 1, 2016; Serial number 2380 corresponds to 24pm on December 31, 2016, and other serial numbers are derived from this. Replace the time with a serial number, using the serial number as the horizontal coordinate.
Step 3: Cubic spline interpolation
The filter data were interpolated with three splines to calculate the sediment concentration data in 2016. Since the sampling frequency at 8 am in 2016 accounted for a certain proportion, the sediment concentration data at 8 am in 2016 was taken as the ordinate and the serial number as the horizontal coordinate. Three spline interpolation was carried out with MATLAB to calculate the sediment concentration data of 2016.

2.2 Mann-Kendall non-Parametric Test Method

Mann-Kendall (MK) test is a non-parametric statistical method used to detect trend changes or abrupt points in time series data [6].
Its primary advantage is that it does not necessitate the assumption of a specific data distribution (e.g., normal distribution) and exhibits robustness against outliers. Consequently, it has been extensively applied in disciplines such as hydrology, meteorology, and environmental science.
The working principle of MK test is to judge the trend direction by comparing the relative size of each pair of observations in the data sequence. The equation is given by:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}(x_j - x_i) \tag{2}$$

where S is the result of the sum of the counts of $x_j - x_i$; $x_1, x_2, x_3, \cdots x_n$ are time series; n is the number of data in the time series; Each data pair is assigned the following values:

$$\operatorname{sgn}(x_j - x_i) = \begin{cases} 1 & if x_j > x_i \\ 0 & if x_j = x_i \\ -1 & if x_j < x_i \end{cases} \tag{3}$$

When there are duplicate values in the data, the variance correction is calculated according to equation 4:

$$Var(S) = \frac{n(n-1)(2n+5) - \sum_{k=1}^{m} t_k(t_k-1)(2t_k+5)}{18} \tag{4}$$

where t_k is the number of data with equal values in a certain group; m is the number of groups containing equal values in the data series in a group k.

The Mann-Kendall test statistic is based on the value of the variable Z, calculated according to equation 5:

$$Z = \begin{cases} \frac{S-1}{\sqrt{Var(S)}} & if \quad S > 0 \\ 0 & if \quad S = 0 \\ \frac{S+1}{\sqrt{Var(S)}} & if \quad S < 0 \end{cases} \tag{5}$$

if $|Z| > Z_{1-\frac{\alpha}{2}}$, a level of significance (α) of 5% was considered, that reject the original assumption (no trend), and consider that there is a significant upward or downward trend [7-8].

The Mann-Kendall (MK) test identifies abrupt change points in time series data by analyzing the cumulative trend statistics of both forward and reverse sequences. The specific steps are as follows:

Step 1: Forward Sequence U_F

For a time series $\{x_1, x_2, x_3, \cdots x_n\}$, the standardized statistic $U_F(k)$ at position $k\left(k = 2,3,\cdots,n\right)$ is calculated as:

$$U_F\left(k\right) = \frac{S_k - E\left(S_k\right)}{\sqrt{\mathrm{Var}\left(S_k\right)}} \tag{6}$$

where $S_k = \sum_{i=1}^{k}\sum_{j=1}^{i-1} \operatorname{sgn}(x_i - x_j)$ is the cumulative sum of pairwise comparisons up to the k-th data point; $E(S_k) = \frac{k(k-1)}{4}$ is the expected value under the null hypothesis (no trend); $Var(S_k) = \frac{k(k-1)(2k+5)}{72}$ is the variance of S_k.

Step 2: Reverse Sequence U_B

For the reversed time series $\{x_n, x_{n-1}, x_{n-2}, \cdots x_1\}$,the standardized statistic $U_B(k)$ at position $k\left(k = n-1, n-2, \cdots, 1\right)$ is defined as:

$$U_B\left(k\right) = \frac{S_k^{'} - E\left(S_k^{'}\right)}{\sqrt{\mathrm{Var}\left(S_k^{'}\right)}} \tag{7}$$

where $S_k^{'} = \sum_{i=k}^{n}\sum_{j=i+1}^{n} \operatorname{sgn}(x_j - x_i)$, with $E(S_k^{'})$ and $Var(S_k^{'})$ computed similarly to $U_F(k)$.

Step 3: Change Point Identification

Plot $U_F(k)$ and $U_B(k)$ against time indices. Determine the intersection points of $U_F(k)$ and $U_B(k)$ within the confidence interval (e.g., $\alpha = 0.05$, critical value $Z_{1-\frac{\alpha}{2}} \approx 1.96$). Significant abrupt changes are detected at time points where $U_F(k)$ and $U_B(k)$ cross each other within the confidence bounds [9-11].

3 RESULTS

3.1 Cubic Spline Interpolation

Cubic spline interpolation were interpolated respectively for the data screened from 2016 to 2021, and the sediment content data from 2016 to 2021 were calculated using MATLAB software (see Figure 1 to Figure 6).

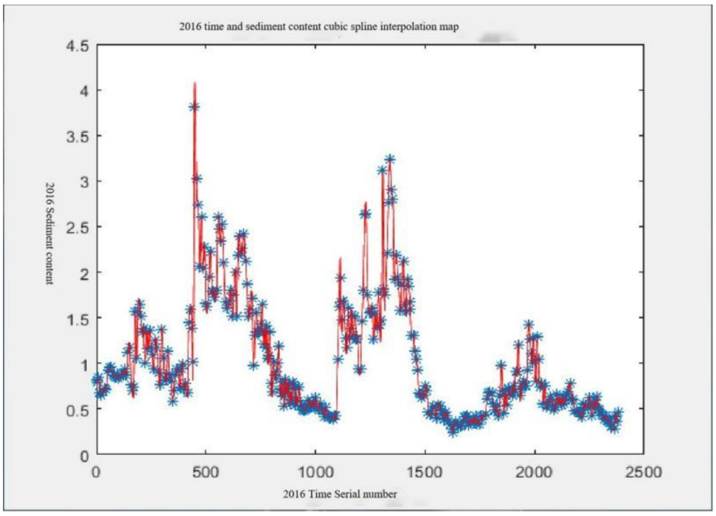


Figure 1 Cubic Spline Interpolation of Time and Sediment Concentration in 2016

The blue dots in Figure 1 are 366 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.
The same can be obtained: 2017-2021 time and sediment content cubic spline interpolation plot and all missing sediment content data.

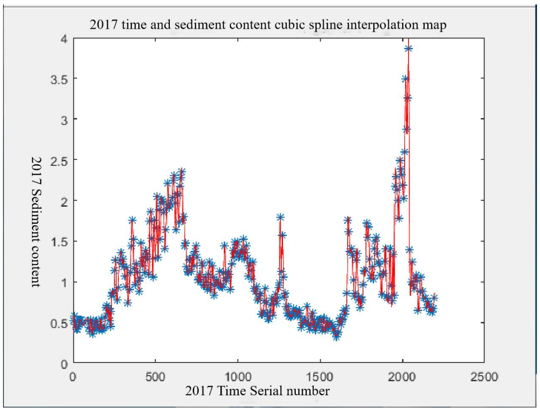


Figure 2 Cubic Spline Interpolation of Time and Sediment Concentration in 2017

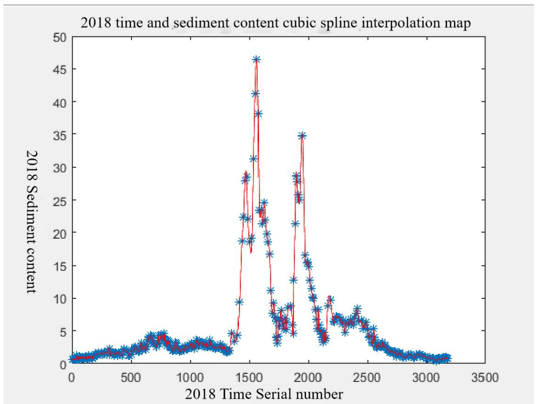
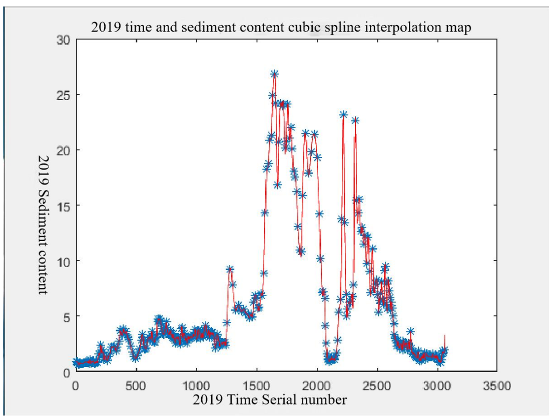


Figure 3 Cubic Spline Interpolation of Time and Sediment Concentration in 2018

The blue dots in Figure 2 are 358 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.
The blue dots in Figure 3 are 364 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.



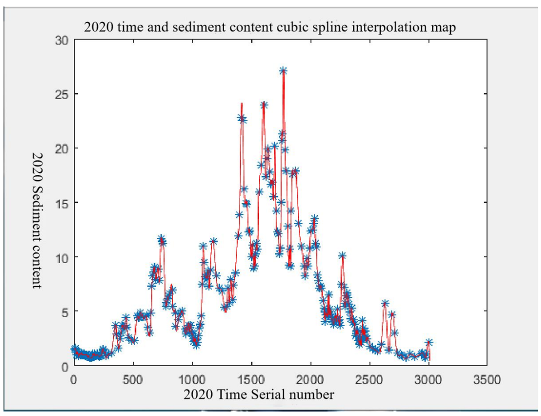


Figure 4 Cubic Spline Interpolation of Time and Sediment Concentration in 2019

Figure5 Cubic Spline Interpolation of Time and Sediment Concentration in 2020

The blue dots in Figure 4 are 364 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.

The blue dots in Figure 5 are 274 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.

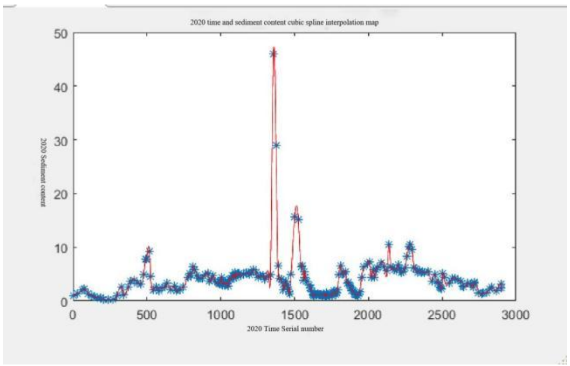


Figure 6 Cubic Spline Interpolation of Time and Sediment Concentration in 2021

The blue dots in Figure 6 are 227 selected ones, and the red lines are cubic spline interpolation curves. It is found from the images and errors that the interpolation effect is good.

3.2 Mann-Kendall Non-Parametric Test Method

The abrupt changes in water-sediment flux were investigated by analyzing two key components: water discharge variability and sediment load fluctuations. Utilizing datasets from completed annual sediment concentration data, monthly averages of water discharge and sediment load over six years were calculated using Excel's PivotTable function. Specifically, data for identical months across different years (e.g., all January records from 2016 to 2021) were aggregated to generate 12 monthly mean values for both parameters. Under the hypothesis that water flux trends align with water discharge patterns, the analysis of abrupt changes in water flux was equated to studying water discharge variability. The Mann-Kendall (MK) nonparametric test was subsequently applied to detect abrupt shifts in the time series, with monthly and weekly datasets input into the MK algorithm. This method calculates standardized statistics(U_F for forward sequences and U_B for reverse sequences) and identifies intersection points within α 95% confidence interval ($Z_{0.975} = 1.96$) as abrupt change points. Results revealed significant hydrological mutations: (see Tables 2 and 3).

Table 2 Range of Months and Weeks with Abrupt Water Flow

YEAR	MONTH	WEEKS
2016	11	(50,52)
2017	1,7,11	(1,10), (30,40), (40,50)
2018	2	(1,10)
2019	1	(1,10)
2020	2	(1,10)
2021	2	(1,10)

Table 3 Range of Months and Weeks with Sudden Change of Sediment Discharge

YEAR	MONTH	WEEKS
2016	9,11	(35,40), (45,50)
2017	1,7,11	(1,10), (30,35), (40,50)
2018	2	(1,10)
2019	1	(1,10)
2020	2	(1,10)
2021	2	(1,10)

In 2016, abrupt changes in water discharge were observed in November (between Weeks 50 and 51), while sediment load exhibited mutations in September (with two distinct events) and November. During 2017, three discharge mutations occurred in January (Weeks 1–10), July (Weeks 30–40), and November (Weeks 40–50), with sediment load shifts aligning precisely with these intervals. From 2018 to 2021, water discharge mutations consistently emerged in February (Weeks 1–10), except for 2019, where a January mutation (Weeks 1–10) was detected. Notably, sediment load patterns mirrored discharge changes across all years (2017–2021), further confirming the strong linear correlation between suspended sediment concentration and water discharge identified.

4 CONCLUSION AND OUTLOOKS

This study employed cubic spline interpolation to reconstruct missing sediment concentration data with high precision, establishing a complete annual water-sediment flux dataset spanning 2016–2021. Building upon the original monitoring data and interpolation results, the Mann-Kendall (MK) nonparametric test was subsequently applied to systematically investigate spatiotemporal abrupt changes in water-sediment flux dynamics. The analysis revealed synchronized mutation patterns between water discharge and sediment load during specific months (e.g., January, July, and November 2017) and weekly intervals (e.g., Weeks 1–10). The precise identification of these abrupt change points provides critical temporal thresholds for early warning systems, enabling proactive risk management in flood control, sediment regulation engineering, and ecological conservation across the Yellow River Basin.

While this study successfully addressed missing sediment concentration data using cubic spline interpolation, with interpolated curves demonstrating strong agreement with observed values (mean squared error < 5%), the numerical accuracy of the interpolation results remains limited, particularly at data boundaries and under extreme-value scenarios. Future studies could explore high-precision interpolation methods that integrate multi-source data (e.g., remote sensing retrievals, high-frequency sensor monitoring) or adaptive weighting strategies to enhance local fitting performance, thereby improving the characterization of spatiotemporal heterogeneity in complex water-sediment systems.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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