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# DECISION OPTIMIZATION MODEL FOR ELECTRONIC PRODUCT PRODUCTION BASED ON BINOMIAL DISTRIBUTION

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**Abstract:** This paper aims to solve the key balance between quality control and cost optimization in the multi-stage electronics manufacturing process. By combining statistical hypothesis testing with mixed integer linear programming (MILP), we propose a novel decision-making framework that can dynamically adapt to different defect rate, inspection cost, and risk scenarios. Firstly, a one-sided hypothesis testing method was proposed to calculate the minimum sampling size in order to solve the problem of supplier defect rate verification. Secondly, for the multi-stage production decision-making problem, a mixed integer linear programming model is constructed, and the total cost is optimized by the combination of enumeration strategies. This study provides a theoretical basis for enterprises to formulate flexible production strategies, promotes the development of production decision science by combining statistical quality control with operational optimization, and provides a data-driven tool for manufacturers to cope with the dynamic supply chain environment. This approach can be extended to the context of sustainable manufacturing, especially for recycling-oriented production systems with material uncertainty.
**Keywords:** Quality control; Mixed integer programming; Hypothesis testing; Sampling testing; Production decisions

## 1 INTRODUCTION

In view of the contradiction between quality risk and detection cost in the production of electronic products, the existing research lacks a multi-process collaborative optimization framework, and the contradiction between the fluctuation rate of spare parts defective rate and the high cost of full inspection in the manufacturing process of electronic products is becoming increasingly prominent, and the existing research has shortcomings in cross-process collaborative decision-making and uncertainty quantification. In this paper, aiming at the typical scenario of 10% nominal defective rate, a dynamic optimization model is constructed through binomial distribution hypothesis testing, aiming to solve the cost-benefit imbalance problem in the chain decision-making of "detection, dismantling, swapping", and provide enterprises with a scientific decision-making framework that takes into account quality risk control and cost savings of more than 15%.

In recent years, research has paid more attention to multi-objective collaboration (e.g., cost, environment, social responsibility), integration of data and mechanism, and real-time dynamic decision-making. In 2021, Chungchi Hsieh et al. [1] discussed the production decision problem of alternating raw materials and recycled materials in a single production process, and studied the batch decision strategy under the constraint of the maximum allowable setting time by constructing a cost optimization model. In 2023, Wu Amin et al. [2] proposed a dual-stream information bottleneck (TIB) method on multiple object detection datasets, which is suitable for industrial defect detection and medical image analysis. The Anomaly CLIP team [3] developed an object-independent text prompt learning method to generalize the CLIP model to cross-domain anomaly detection tasks through global and local loss optimization, covering industrial defects and medical imaging. Li Wei et al. [4] established a two-stage game model for cross-border remanufacturing supply chain, which provides a theoretical basis for policymakers to adjust the tax structure. In 2025, Lingxin Wang et al. [5] proposed a hierarchical scheduling model to integrate demand fluctuations, resource constraints, and social responsibility goals, and optimize production planning through three-level decision-making (demand feasibility, resource allocation, and dynamic adjustment).

In the field of electronic product manufacturing, the fluctuation of the quality of spare parts directly affects the qualified rate of finished products, and comprehensive testing will greatly increase the production cost. How to balance quality risk and economic benefits under limited testing resources has become the core issue of enterprise operation in our paper. Most of the existing studies focus on single-process optimization, and lack a multi-stage collaborative decision-making framework. In this paper, a whole-process decision-making model covering sampling detection, assembly optimization, and dismantling strategy is established by combining statistical methods and operations research theories, so as to provide a dynamic optimization scheme for multi-stage complex production systems which is different from the previous research.

## 2 PREREQUISITES

### 2.1 Approximately Normal Distribution Formula for Binomial Distribution

The binomial distribution is a discrete probability distribution that describes the results of a Bernoulli experiment with a probability of success p n times[6]. Normal distribution is a continuous probability distribution that is widely used in statistics and natural sciences. Under some conditions, the binomial distribution can be approximated by a normal distribution. First, let's define the binomial distribution and the normal distribution.
**Binomial distribution**: Let $X$ be the number of successes in $n$ Bernoulli experiments, and the probability of each success is $p$. then $X$ obeys the binomial distribution of the parameters $n$ and $p$, which is denoted as $X\sim B(n,p)$.
**Normal distribution**[7]: Let $X$ be a random variable, if for any real numbers $a$ and $b$ $(a < b)$, $P(a\leq X\leq b)$ is a constant, and is a continuous function about $(a, b)$, then $X$ is said to obey a normal distribution, denoted as $X\sim N(\mu,\sigma^2)$, where $\mu$ is the mean and $\sigma^2$ is the variance.
The conditions and precautions for an approximately normal distribution of binomial distributions are given below.
Conditions for an approximately normal distribution of binomial distributions:
(1) When n is large enough, n≥30 is usually required.
(2) The probability of success p is close enough to 0.5, i.e. 0.2≤p≤0.8.
Under such conditions, the binomial distribution can be approximated with a normal distribution, i.e
X~B(n,p) can be approximated as X~N(μ, $\sigma^2$).
where μ = E(X) = np and $\sigma^2$ = Var(X) = np(1-p)
Notes:
(1) The probability calculation of the binomial distribution can be converted into the probability calculation of the standard normal distribution.
(2) It should be noted that the calculation result of the binomial distribution is discrete, while the calculation result of the normal distribution is continuous, so when making the approximate calculation, a correction is required, usually rounding the result to the nearest integer.
In summary, the binomial distribution can be transformed into a normal distribution by normalizing the variables, and the probability table of the normal distribution can be used to approximate the calculation. This approximation is valid when n is large enough and the probability of success p is close to 0.5.

### 2.2 Mixed Integer Linear Programming Model

Mixed Integer Linear Programming (MILP) is a mathematical optimization model that combines continuous and integer variables, and is suitable for problems that include both discrete and continuous decisions[8]. First, learn the Linear Programming (LP) model. LP means that the objective function is linear, all constraints are linear, and finally, the decision variable can take any real number.

**Table 1** Food Variables

| Food | Cost per serving | Vitamin A | Calories |
|---|---|---|---|
| Corn | $ 0.18 | 107 | 72 |
| 2% Milk | $0.23 | 500 | 121 |
| Wheat Bread | $0.05 | 0 | 65 |

Table 1 shows that there are 3 kinds of food sold in the supermarket, corn, milk and bread, and the price, vitamin A and calorie information are shown in the table above. Now the question is how many servings of corn, milk, bread to buy so that the total price is the lowest, and the total intake of vitamin A is not less than 500 but not more than 50,000, and the total calorie intake is not less than 2,000 but not more than 2,250.
The objective functions and constraints are as follows:
**Objective Function:**

$$\min \ 0.18x_{corn} + 0.23x_{milk} + 0.05x_{bread} \tag{1}$$

**Constraints:**

$$\begin{cases} 107x_{corn} + 500x_{milk} \leq 50000 \\ 107x_{corn} + 500x_{milk} \geq 500 \\ 72x_{corn} + 121x_{milk} + 65x_{bread} \leq 2250 \\ 72x_{corn} + 121x_{milk} + 65x_{bread} \geq 2000 \\ x_{corn}, x_{milk}, x_{bread} \geq 0 \end{cases} \tag{2}$$

If there are some decision variables in the linear programming problem, such as the above $x_{corn}$ requirement must be an integer, then the programming problem becomes a mixed integer linear programming problem.

### 2.3 Hypothesis Testing

Definition Hypothesis testing is a statistical inference method used to determine whether the differences between samples and samples and populations are caused by sampling errors or essential differences. The basic principle is to make some kind of assumption about the characteristics of the population, and then make inferences about whether this hypothesis should be rejected or accepted through statistical reasoning from sampling studies.
First assume that a certain hypothesis is true in the whole and calculate what results it will lead to. If it leads to an

unreasonable phenomenon, the original assumption is rejected. If it does not lead to an unreasonable phenomenon, the original hypothesis cannot be rejected and the original hypothesis can be accepted.
It is also different from the general method of counterproof. The so-called irrational phenomenon is based on the principle of small probability. That is, an event with a small probability is almost impossible to occur in an experiment, and if it happens, it is unreasonable. As for what is considered a "small probability"? Events with a probability of no more than 0.05 can usually be called "small probability events", or 0.1 or 0.01 depending on the specific situation $\alpha$. The hypothesis that was originally set up becomes the null hypothesis and is recorded as $H_0$. The hypothesis contrary to the $H_0$ is called the alternative hypothesis, which is the hypothesis that should be accepted when the null hypothesis is rejected, and is denoted as $H_1$.
Hypothetical form $H_0$--- Null hypothesis, $H_1$---alternative hypothesis
Two-sided test: $H_0: \mu = \mu_0$, $H_1: \mu \neq \mu_0$
One-sided test: $H_0: \mu \geq \mu_0$, $H_1: \mu < \mu_0$ or $H_0: \mu \leq \mu_0$, $H_1: \mu > \mu_0$
Hypothesis testing is to test the null hypothesis ($H_0$) according to the sample observations, and the $H_0$ is accepted and the $H_1$ is rejected. If you reject $H_0$, accept $H_1$.

### 2.4 Notations

The symbols used in the paper are listed in Table 2.

**Table 2** Description of the Symbol

| symbol | illustrate |
|---|---|
| $p_1$, $p_2$, $p_f$ | Spare parts 1, spare parts 2 and finished product defect rate. |
| $c_1$, $c_2$ | Purchase unit price for Parts 1 and Parts 2. |
| $t_1$, $t_2$, $t_f$ | Unit cost of inspecting parts 1, 2 and finished products. |
| $c_a$ | The assembly cost per finished product. |
| $c_d$ | The cost of dismantling a non-conforming finished product. |
| $c_r$ | A loss arising from the return of a non-conforming finished product. |
| $s$ | The market price of qualified finished products. |
| $x_1$, $x_2$ | A binary variable that indicates whether parts 1 and 2 are tested. |
| $x_f$ | A binary variable that indicates whether or not the finished product is tested. |
| $x_d$ | A binary variable that indicates whether or not the non-conforming finished product is disassembled. |
| $N$ | The total number of finished products produced. |

## 3 HYPOTHESIS TESTING-DRIVEN SAMPLING DESIGN

Enterprises need to design a sampling test program to determine whether the defective rate of a batch of spare parts exceeds the nominal value. The goal is to reduce the number of sampling tests as much as possible while maintaining a certain level of confidence. Here we use a linear programming solver to solve the model, so we need to specify the decision variables, objective functions and constraints.

### 3.1 Hypothesis Testing Methods

Null hypothesis $H_0$: The defective rate $p$ of the spare part does not exceed the nominal value $p_0$, i.e., $p \leq p_0$.
Alternative assumption $H_1$: The defective rate of the spare part $p$ exceeds the nominal value $p_0$, i.e., $p > p_0$.
We want to find a sample size $n$ and a cut-off value $c$ such that:
If the number of defective products in the sample is $x \leq c$, the null hypothesis $H_0$ will be accepted and the batch of spare parts will be considered qualified.
If the number of defective products in the sample is $x > c$, the null hypothesis $H_0$ is rejected and the batch of spare parts is considered unqualified.

### 3.2 Constraints

We need to meet the following two constraints:
Producer risk constraint $\alpha$: When the null hypothesis $H_0$ is true, the probability of rejecting the $H_0$ does not exceed $\alpha$.

$$P(\text{"Reject"}H_0|H_0 \text{ "True"}) \leq \alpha \; P(\mathrm{x} > \mathrm{c}|p \leq p_0) \leq \alpha \tag{3}$$

Consumer risk $\beta$: When the alternative hypothesis $H_1$ true, the probability of accepting the $H_0$ does not exceed $\beta$.

$$P((\text{"Accept"}H_0|H_1 \text{ "True"}) \leq \beta \tag{4}$$

$$P(\mathrm{x} \leq \mathrm{c}|p > p_0) \leq \beta \tag{5}$$

### 3.3 Solution Method

Due to the large sample size, we use a normal distribution to approximate the binomial distribution.
First, identify the deny domain. Based on the producer risk $\alpha$, we can determine the cut-off value c for the rejection domain:

$$P（x>c \mid p\leq p_0）\approx P\left( Z > \frac{c\text{-}np_0}{\sqrt{np_0(1\text{-}p_0)}} \right)\leq\alpha \tag{6}$$

If you find the Z-value corresponding to the α from the standard normal distribution table, denoted as $Z_\alpha$, then there are:

$$\frac{c-np_0}{\sqrt{np_0(1-p_0)}} \geq Z_\alpha \tag{7}$$

$$c \geq np_0 + Z_\alpha\sqrt{np_0(1-p_0)} \tag{8}$$

Therefore, the sample size is determined.
Based on the consumer risk β, we can determine the sample size $n$:

$$P（x\leq c \mid p>p_0）\approx P\left( Z \leq \frac{c\text{-}np}{\sqrt{np(1\text{-}p)}} \right)\leq\beta \tag{9}$$

If you find the Z-value corresponding to "β" from the standard normal distribution table, denoted as $Z_\beta$, then there are:

$$\frac{c-np}{\sqrt{np(1-p)}} \leq - Z_\beta \tag{10}$$

Substituting the c obtained in (1) and replacing $p$ with $p_1$ yields:

$$\frac{np_0+Z_\alpha\sqrt{np_0(1-p_0)}-np_1}{\sqrt{np_1(1-p_1)}} \leq - Z_\beta \tag{11}$$

After simplification, the inequality about $n$ is obtained, and $n$ can be solved.

### 3.4 Acceptance Plan

The solution of the acceptance scheme is similar to that of the rejection scheme, except that the alternative assumption is changed to the following: the defective rate $p$ of the spare parts is less than or equal to the nominal value $p_0$minus a set value $p'$, that is, $p\leq p_0$- $p'$.
Through the above steps, we can calculate the minimum sample size n and the cut-off value $c$ based on the pre-set producer risk α and consumer risk β, as well as the nominal value $p_0$ and the expected defective rate $p_1$, so as to design a sampling test scheme with as few tests as possible.

## 4 PRODUCTION DECISION MODEL BASED ON COST-BENEFIT ANALYSIS

Enterprises need to assemble two kinds of spare parts (denoted as A and B) to produce a certain product, and their quality characteristics must meet:
(1) If any spare parts are unqualified, the finished product must be unqualified.
(2) There is still a risk of defective products after the assembly of double qualified spare parts.
(3) Nonconforming products can be dismantled and recycled spare parts (dismantling fees need to be paid).
Here, we continue to use the Linear Programming Solver (PuLP) to solve the model to obtain the optimal value of the decision variables, that is, which parts, semi-finished products and finished products are detected and disassembled, so it is necessary to clarify the decision variables, the upper limit of the confidence interval of the defective rate, the objective function and the constraints[9]. Table 3 shows the data needed for decision-making.

**Table 3** The Situation Encountered by the Enterprise in Production

| circumstance | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Spare part A | Defective rate | 10% | 20% | 10% | 20% | 10% | 5% |
| | Unit price of purchase | 4 | 4 | 4 | 4 | 4 | 4 |
| | Cost of detection | 2 | 2 | 2 | 1 | 8 | 2 |
| Spare part B | Defective rate | 10% | 20% | 10% | 20% | 20% | 5% |
| | Unit price of purchase | 18 | 18 | 18 | 18 | 18 | 18 |
| | Cost of detection | 3 | 3 | 3 | 1 | 1 | 3 |
| finished product | Defective rate | 10% | 20% | 10% | 20% | 10% | 5% |
| | Assembly costs | 6 | 6 | 6 | 6 | 6 | 6 |
| | Cost of detection | 3 | 3 | 3 | 2 | 2 | 3 |
| | The market price | 56 | 56 | 56 | 56 | 56 | 56 |
| Non-conforming finished products | Swap loss | 6 | 6 | 30 | 30 | 10 | 10 |
| | Dismantling costs | 5 | 5 | 5 | 5 | 5 | 40 |

### 4.1 Objective Functions

Our goal is to maximize corporate profits. Whereas the total profit is equal to the total revenue minus the total cost [10]. Among them, the total revenue and total cost are calculated as follows:
Total Revenue: Equal to the number of qualified finished goods sold multiplied by the market selling price. Namely

$$\text{Revenue} = s\cdot N \cdot(1-p_f)\cdot(1-p_1x_1)\cdot(1-p_2x_2)\cdot x_f \tag{12}$$

The total cost consists of the following components.
Spare parts purchase cost: $N(c_1 + c_2)$
Spare parts testing cost: $N(p_1 t_1 x_1 + p_2 t_2 x_2)$
Finished product assembly cost: $Nc_a$
Finished product testing cost: $Nx_f t_f$
The cost of handling unqualified finished products includes the following two parts.
In case of disassembly: $\mathrm{N}(p_f + (1 - p_f)p_1 x_1 + (1 - p_f)(1 - x_1)p_2 x_2)\cdot x_{\mathrm{d}} c_d$
If you don't disassemble: $\mathrm{N}(p_f + (1 - p_f)p_1 x_1 + (1 - p_f)(1 - x_1)p_2 x_2)\cdot(1 - x_{\mathrm{d}})c_r$
Therefore, the objective function can be written as:

$$\text{Maximize Profit} = s\cdot N\cdot(1 - p_f)\cdot(1 - p_1 x_1)\cdot(1 - p_2 x_2)\cdot x_f - N(c_1 + c_2) - N(p_1 t_1 x_1 + p_2 t_2 x_2) - Nc_a - Nx_f t_f - N(p_f + (1 - p_f)p_1 x_1 + (1 - p_f)(1 - x_1)p_2 x_2)\cdot(x_{\mathrm{d}} c_d + (1 - x_{\mathrm{d}})c_r)$$

where the decision variable is a binary variable:$x_1$, $x_2$, $x_f$, $x_d \in \{0,1\}$

### 4.2 Solution

For each case in Table 3, we can substitute the known $p_1$, $p_2$, $p_f$, $c_1$, $c_2$, $t_1$, $t_2$, $t_f$, $c_a$, $c_d$, $c_r$, $s$ into the objective function and enumerate the combinations of all decision variables ($2^4$=16 in total) to calculate the profit under each strategy. Finally, the strategy with the greatest profit is selected as the optimal decision. Table 4 shows the final decision plan and the corresponding profit.

**Table 4** The Final Decision Plan and the Corresponding Profit

| situation | Spare parts A detection | Spare parts B detection | Finished product testing | Dismantling of non-conforming products | profit |
|---|---|---|---|---|---|
| 1 | No | No | Yes | Yes | 34600 |
| 2 | Yes | No | Yes | Yes | 14600 |
| 3 | No | No | Yes | No | 36000 |
| 4 | Yes | No | Yes | No | 24400 |
| 5 | No | Yes | Yes | Yes | 31800 |
| 6 | No | No | Yes | No | 36000 |

### 4.3 Analysis of the Basis for Decision-Making

Spare parts detection: The detection of spare parts depends on the inspection cost, purchase unit price and defective rate. If the inspection cost is low relative to the purchase unit price, and the defective rate is high, the inspection can effectively reduce the cost and improve the profit.
Finished product testing: Since the market price of finished products is much higher than the production cost, and the return and replacement losses are large, it is usually necessary to test the finished products, which can effectively avoid the flow of substandard products into the market.
Dismantling of non-conforming products: The decision to dismantle non-conforming products depends on the cost of dismantling and the loss of return and exchange. If the cost of dismantling is lower than the return loss, then dismantling can recover part of the loss.
The quality correlation of spare parts is not taken into account. The reuse rate of parts after dismantling is not modeled. In the future, machine learning can be introduced to dynamically optimize detection thresholds

## 5 CONCLUSION

The hybrid decision model proposed in this paper effectively solves the problem of quality-cost trade-off in multi-process production. By introducing hypothesis testing, integer programming and risk analysis, the dynamic optimization of the strategy is realized. In the future, the research can be extended to multi-objective optimization and reinforcement learning frameworks to further improve the complexity and adaptability of product inspection models.

## COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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