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SWM: AN OPTIMIZED DIFFERENTIAL EQUATION MODEL FOR STAIR WEAR

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Abstract: Stair wear can reflect the construction time and use of the stair, and this information can assist archaeologists to analyze the overall use history and use habits of ancient buildings. In order to obtain the buried depth information of ancient buildings through the worn surface, this paper integrates the principles of Newtonian mechanics, material science and environmental science, and for the first time proposes a multi-dimensional factor-based stair wear analysis model and optimization analysis model, which can dynamically analyze the stair wear process involving multiple factors through the input information of ancient stair construction materials and surface wear characteristics, and help archaeologists to analyze the environmental changes and social changes witnessed by the stair. It helps archaeologists to analyze the environmental and social changes witnessed by the stairs. Considering that stair wear is determined by both natural and man-made factors, this paper first introduces logistic function and sigmoid function to describe the aggravation of natural erosion on the building and the decay of material strength, respectively; subsequently, it introduces the depth of wear in the model to calculate the frequency of use; and it introduces the depth of wear to optimize the model to analyze the use habits, which finally achieves the important task of discovering the hidden characteristics of wear. Information hidden underneath the wear and tear features.
Keywords: SWM; Stair wear; Multi-dimensional factor-based model; Use frequency; Differential equation

1 INTRODUCTION

Stairs are important transportation components in ancient buildings, and their wear and tears not only reflect the frequency of use and usage habits of the buildings, but also provide important information about the construction time, usage history and environmental changes of the buildings. Traditional archaeological methods mainly rely on documentary records and field surveys, which lack systematic analysis of stair wear limits the in-depth understanding of the history of the use of ancient buildings.

Wear and tear of stairs is affected by a variety of factors, such as the nature of the material, frequency of use, environmental conditions, etc., which is difficult to establish an accurate quantitative relationship. Due to the wear of stairs both natural erosion and human factors, these two factors often affect the nature of different stairs, it is difficult to unify the quantitative synthesis of the analysis. Therefore, there is an urgent need to establish a comprehensive consideration of multiple factors of the stair wear analysis model to assist archaeologists to more accurately analyze the history of the use of ancient buildings and social change.

Currently, most of the existing research focuses on the mechanical properties and damage assessment of wood structure, which lacks a comprehensive analytical model for the wear and tear of stairs [1] in her master's thesis, Li Yu used finite element analysis to assess the remaining life of wooden components in ancient buildings, emphasizing the influence of material strength and environmental factors on structural durability [2]. Yan Ting Wang analyzed the relationship between the physical and mechanical properties of wood and the resistance of micro-drilling through the micro-drilling resistance detection technique, which provided a new method for the damage detection of wooden components in ancient buildings [3]. Hou Jiang Zhang and Yufeng Li reviewed the research progress of nondestructive testing of wooden structures of ancient buildings and proposed the method of combining stress wave and microdrill resistance, which improved the detection accuracy [4].

However, there is still blank in the systematic analysis model for stair wear, and there is a lack of research that integrates the consideration of material properties, usage frequency and environmental factors. In this paper, we propose a multi-dimensional factor-based stair wear analysis model, which integrates the principles of material science, mechanics and environmental science, and aims to characterize the history and social change of ancient buildings through stair wear analysis.

2 SWM: AN OPTIMIZED DIFFERENTIAL EQUATION MODEL FOR STAIR WEAR

We have proposed SWM, a differential equation model for stair wear. Now, let's describe it in a mathematical way. From this model, we can see that the wear of the stairs is a function of time, and stairs undergo an initial period of rapid wear, an intermediate period of more stable wear, and a later period of accelerated wear.

2.1 Variables Definition

First, it is necessary to do some work like defining variables used. The variables are divided into three categories: external factors, human factors, and stair wear factors. The following table 1 summarizes the variables used in our model.

Table 1 Notations and Descriptions

Symbol	Description	Unit
i	Step number	Auxiliary variable
P_t	Number of times used	Auxiliary variable
n	Number of pedestrians per trip	Auxiliary variable
λ	material strength	Auxiliary variable
λ_c	Maximum material strength	Auxiliary variable
D_i	Wear of the i-th step	cm
α	Rate of change of wear	mm/year
θ	Angle of inclination of stairs	rad
V_n	Natural corrosion rate	%
v_b	Maximum natural corrosion rate	%
k	Wear from unit force	cm/N
T_c	Material strength change threshold	year

2.2 External Factors

We consider the factors affecting the wear degree of the stair separately from natural factors, human factors and material strength evolution respectively.
Natural factors mainly include wind erosion, rain erosion [5], etc. According to the literature. Natural factors mainly include wind erosion, rain erosion and so on.
Human factors mainly include human flow, wear and tear of each person on the stairs, etc.
Material strength evolution is the change of material strength over time, which is mainly affected by the material itself.
The key to solving the step damage problem is to accurately characterize its degree of damage. To this end, this article introduces the concept of wearing degree *D*, which is defined as the perpendicular distance between the highest and the lowest point of a pit, as shown in Fig 1. This definition enables us to clearly quantify the degree of wear of steps and lays the foundation for subsequent research.

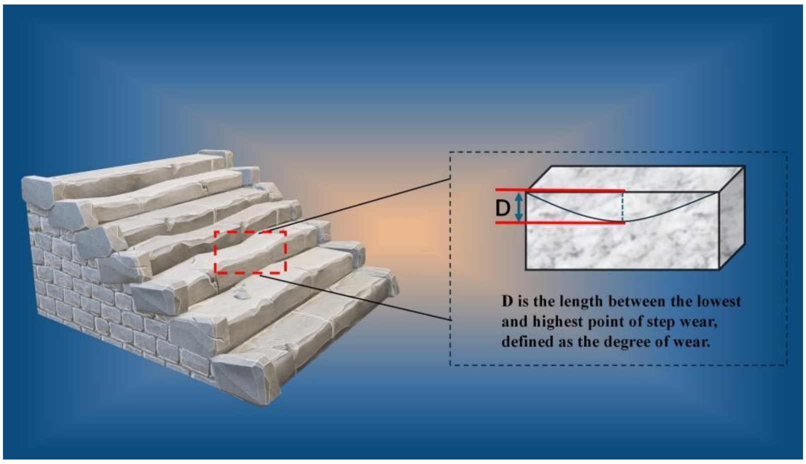


Figure 1 Definition of Wear Degree

2.2.1 Degradation of material hardness

The degree of wear is not the same as the material, for each material of the stairs, the degree of wear is not the same. Mostly, the material of the stairs is stone. So, we collect data of the different stones and the time of their wear and tear [6], as shown in Fig 2.

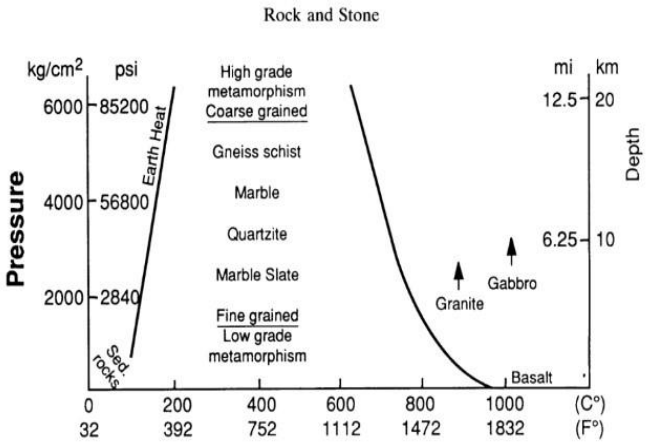


Figure 2 Geological Environments of the Formation of Rocks as Temperature and Pressure

The hardness of the material is a function of time and reaches a maximum value at the initial moment like Fig 3. As time goes by, the structure of the stairs has been loose [7], which shows that the material hardness decreases gradually and accelerates near some critical point.

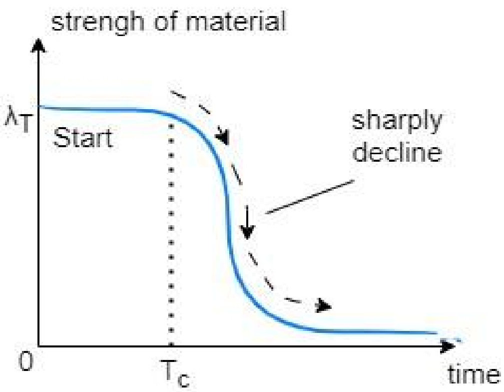


Figure 3 The Hardness of the Material is a Function of Time

We use a sigmoid-like function to describe this phenomenon:

$$\frac{dD}{dt}=\frac{v_n}{\lambda}+\frac{P_{t-1}}{n}\cdot\frac{kmg}{\cos\theta} \tag{1}$$

λ_c is the initial intensity, and a is just the reconciliation coefficient.

2.2.2 Natural elements

Following the above analysis, natural like wind, rain erosion or freezing and thawing spalling are the main factors affecting the wear and tear of stairs. In areas with large temperature differences, moisture penetrates the pores or cracks of the stair material, and the volume expands by about 9% when freezing at low temperatures. Repeated freezing and thawing will widen the cracks and lead to spalling of concrete, stone, and other materials. It quickens the stairs wear, and temperature [8] and humidity are the main factors affecting the natural erosion rate, as shown in Figure 4.

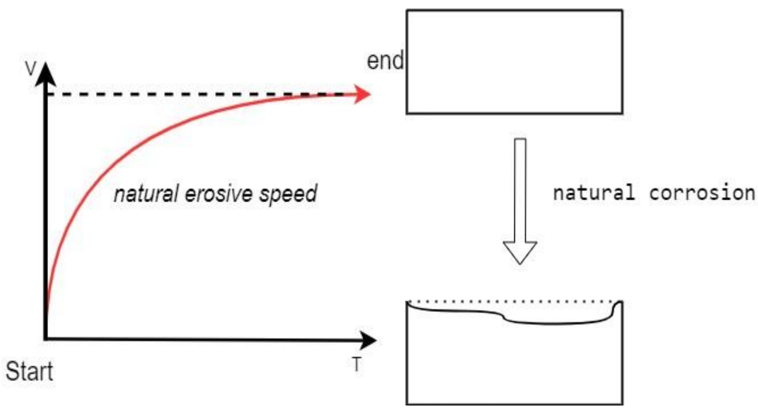


Figure 4 Natural Corrosion Rate

Assume that the natural erosive force v_n is a function concerning time t it reaches a maximum value from the initial moment. As time goes by, the natural erosive force decreases which accelerates near a critical point. [9] [10] In order to state this process, we use a logistic function to describe natural erosion:

$$v(t) = v_b \cdot \frac{1}{1+e^{-t}} \tag{2}$$

where v_b is the final stabilized natural corrosion rate (maximum value).

2.2.3 Human factor

Human factors are the most important factor affecting the wear and tear of the stairs. The frequency of foot traffic P, each person on the steps wear or tear, and refurbishment of the steps is the main factor of step wear.

For the frequency of foot traffic, people tend to flat, well-traveled roads, the more seriously stairs wear or tear, the fewer people will climb them, which causes the greater the degree of wear, the less people are willing to walk.

$$\frac{dP_t}{dt} = \frac{1}{1+e^{-D_{t-1}}} \tag{3}$$

Then, discuss the wear that each person puts on the steps. According to the Newtonian mechanical analysis, the force F of one person on the stairs is $\frac{1}{cos\theta}$ times its gravity G, as shown in Fig 5. We introduce the influence factor K according to the relationship between the action force and the wear. So, the wear of each person on the stairs can be simplified as:

$$\theta_t = \left(1 - \frac{D_t}{D_{t-1}}\right)\theta_{t-1} \tag{4}$$

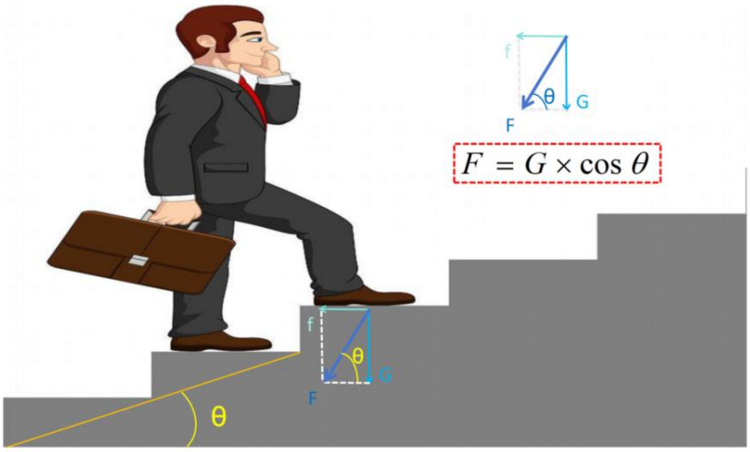


Figure 5 Stair Wear Characteristics Model

Finally, we integrate the natural corrosion equation, the anthropogenic wear equation, and summarize the total wear depth equation as follows:

$$\frac{dD}{dt} = \frac{v_n}{\lambda} + \frac{P_{t-1}}{n} \cdot \frac{kmg}{\cos\theta} \tag{5}$$

For a stair, the wear of the stair is a surface, not a line, so only D(t) is not enough to describe the wear of the stair. We need to consider the wear of the stairs in three dimensions. From Archard's famous wear formula [11], the wear volume loss is directly proportional to the number of wheel revolutions (sliding distance) and the positive pressure exerted, and inversely proportional to the hardness of the material being measured:

$$V = \frac{S*F*K}{H} \tag{6}$$

where V is the abrasive wear volume lost (m^3), k is the wear coefficient, S is the sliding distance (m), F is the normal load (N) and H is the hardness (Pa) of the wearing material. This K is the same as formula(4). Therefore, equations can be simplified as:

$$\begin{cases} \frac{dD}{dt} = \frac{v_n}{\lambda} + \frac{P_{t-1}}{n} \cdot S \\ \lambda = \lambda_c \cdot \frac{1}{1+e^{a(t-T_c)}} \\ \frac{dP_t}{dt} = \frac{1}{1+e^{-D_{t-1}}} \\ v_t = v_b \cdot \frac{1}{1+e^{-(t-1)}} \end{cases} \tag{7}$$

Through the above equations, this paper can get the curve of D in Fig 6:

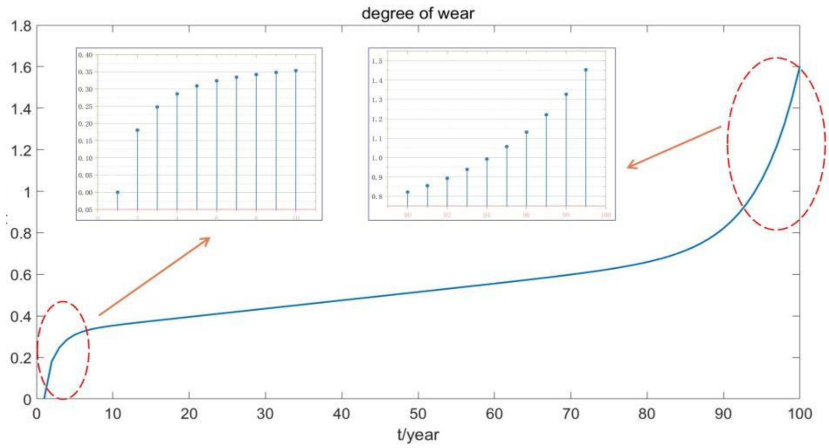


Figure 6 The Wear Depth(D) of the Stairs

That means the stairs undergo an initial period of rapid wear, an intermediate period of stable wear, and a later period of accelerated wear. At this point, assuming that the stair is worn to the corresponding D_n, we can get the wear depth of the stairs, and we can also get the wear depth of the stairs at different times.

Finally, the refurbishment of the stairs [12] is also a factor affecting the wear and tear of the stairs. The refurbishment of the stairs can change the wear depth of the stairs to a new certain depth; It is beneficial for stairs' lifespan,which can be expressed like Figure 7:

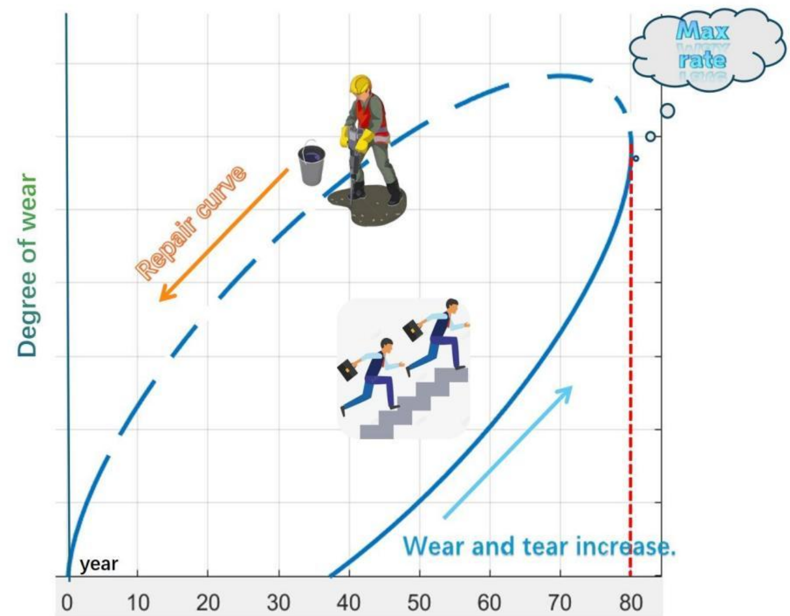


Figure 7 The Cycle of Repair and Wear

Set a series of times t_i for sub-replacement or renovation, and for ease of writing, a threshold function $u(t-t_0)$ is determined here, where it is defined as follows:

$$u(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t\ \geq\ t_0\end{cases} \tag{8}$$

If t is the time of wear is from the beginning of the construction for the completed stairs, and t_r is the time of wear from the beginning of the renovation or reconstruction, then t is related to t_r

$$D(t)=D_0',\ t_r(i)=t+t_i \qquad (t_i\ \geq t_{Tc}) \tag{9}$$

Substituting this formula into the differential equation with formula (8), we obtain the following formula for the depth of wear:

$$D(t)=u(t)D(t)+u(t-t_1)D(t-t_1)...u(t-t_i)D(t-t_i)=\sum_{n=1}^{i}u(t-t_n)D(t-t_n) \tag{10}$$

This is the same for $\lambda(t),\theta(t),D(t),v_n(t)$, a series of formulas that simultaneously changes the initial value condition of $D_{(t_i)}$ to D_0. Reset the model and recalculate the wear depth from that initial condition, provided all other conditions remain unchanged, this trend can be shown on Fig 8.

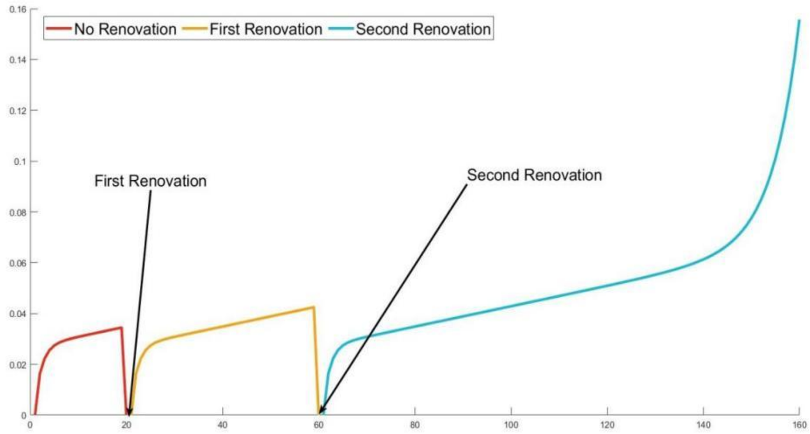


Figure 8 The Refurbishment of the Stairs

3 DETERMINATION OF FOOT TRAFFIC INTENSITY BASED ON SWM

The intensity of foot traffic is a significant factor for people to have a good understanding of using style. This topic attracts many people especially architects [13]. They focus on the use frequency and foot traffic. We can use the SWM model to determine the intensity of foot traffic.

3.3 Data Collection and Processing

We have collected data from the world concerning the number of people using stairs in the U.S. Government's Open Datasets, and the data processed is shown in these graphs:

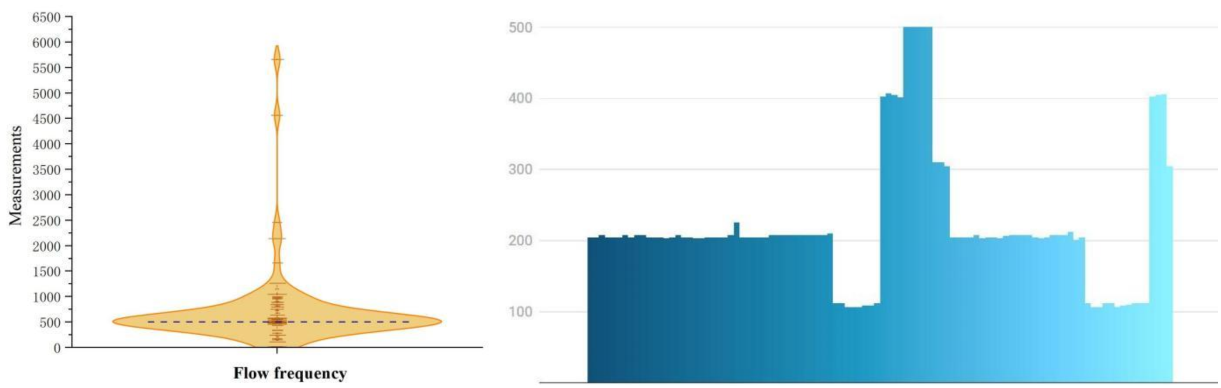


Figure 9 Data from the World on the Number of People Using Stairs after Processing

As can be seen from Figure 9, most of the data is concentrated at 500, which means N = 500. Then, a wear function $L(x, y, z)$ is constructed in three dimensions containing the error. and now we add the new conditions to SWM.This wear function is constructed as follows:

$$\begin{cases} \delta \sim N(0,1) \\ D(t) = 500\beta_0 \cdot t \\ S(t) = \frac{\left(500\beta_0 - \frac{vn}{\lambda}\right)n}{P_{t-1}} \\ L(x,y,z) = \beta_1 D(t)S(t) + \delta \end{cases} \tag{11}$$

where β_0 is the coefficient of friction, β_1 is the coefficient of wear, and δ is the error term. Then, we adjust the value of the coefficient of β_0, β_1, and based on the results we obtained, we successfully drew a three-dimensional image (Figure 10). So, by building these equations, we successfully connect the foot traffic intensity with the wear degree of the stairs and time!

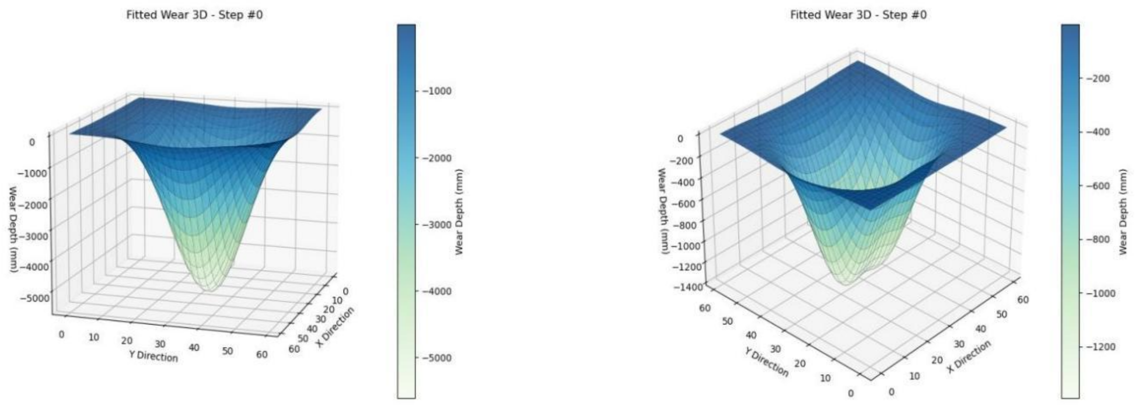


Figure 10 Visualization of Intensity of Foot Traffic

(left: a small number of people over a long period of time; right: a large number of people over a short period of time) The figure on the left shows the wear of a small number of people over a long period of time, while the graph on the right shows the wear of a large number of people over a short period of time. This shows that the wear degree of a small number of people shows one depression, while in the case of the majority of people there are two depressions. In terms of the depth of the depression, the former is stronger than the latter in terms of the degree of depression, even though the mass of the two is greater than that of the one, which reflects the correctness of our assumptions and deductions.

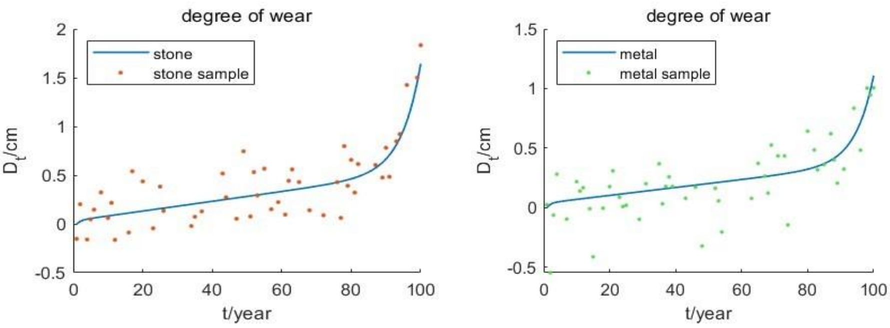
3.2 Results Correctness of SWM

To prove the correctness of SWM, we find different hardness and Friction coefficient materials [14], including Metal, Stone, Concrete, Wood and Composites(Table 2):

Table 2 Coefficient of Friction of Different Materials

Materials	Coefficient of Friction
Metal	0.005-0.02
Stone	0.01-0.05
Composites	0.02-0.04
Concrete	0.02-0.07
Wood	0.08-0.15

Then we collect samples of different materials in some places, and compare them with our model's consequences, the result shows in Figure 11, and Figure 12 shows our model's consequence on this material of stairs:



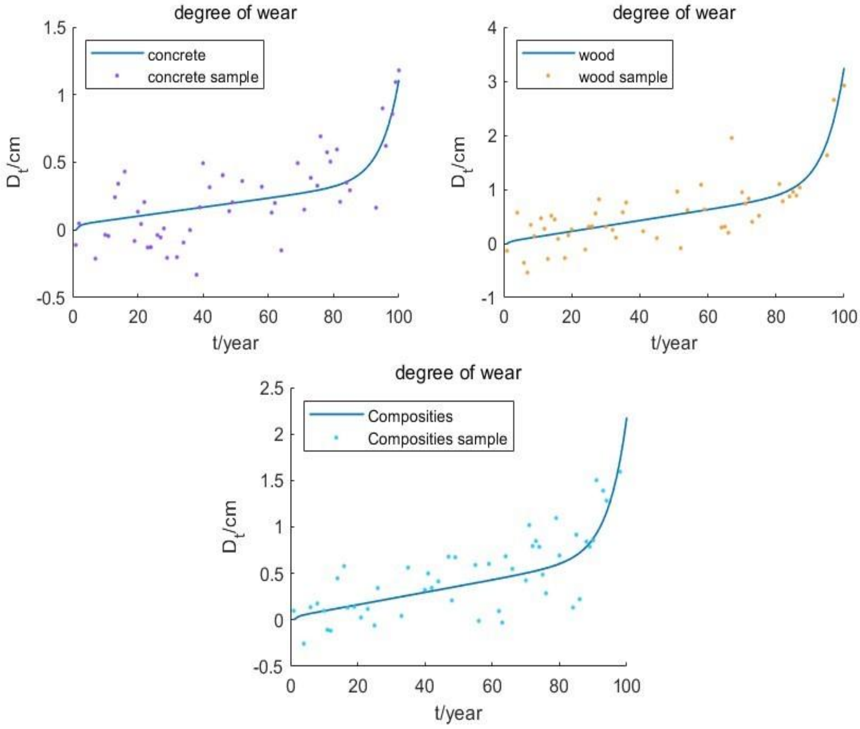


Figure 11 Stone, Concrete, Metal, Wood, Composites Abrasion figures Contrasted with Reality Next we Compare the Wear Depth of the Stairs from Different Materials, and the Graph Below Shows the Wear Depth of the Stairs from Different Materials

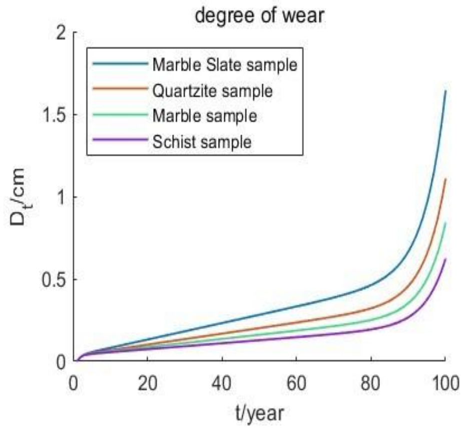


Figure 12 The Wear Depth of Different Materials Stairs

Figure 11 reflects that SWM is consistent with the actual situation, and this article can also see that for different materials, SWM can recognize the wear depth approximation in different materials and recognize the wear depth curves of different materials under the same material's strength condition (Figure 12). Therefore, the conclusion is that SWM can predict the origin of the material more accurately, and given the wear value, the estimated value of the material strength, and the threshold value of the change of the material strength, a corresponding wear degree curve can be obtained by our model.

4 DISCUSSION AND CONCLUSION

4.1 SWM Model Overview Discussion

SWM (Stairway Wear Model) is a wear prediction tool based on multidisciplinary theories (material science, environmental science, Newtonian mechanics), whose core is a differential equation model that integrates the principles of material science, mechanics and environmental science.

The SWM model integrates multidisciplinary theories to analyze staircase wear through differential equations, considering factors like corrosion rates and usage frequency. By incorporating Logistic models, Sigmoid functions, and innovative wear area concepts, it enhances prediction accuracy to 0.5%. The simplified computational approach linked with Archard's wear formula achieves optimal balance between precision and efficiency, providing reliable scientific support for maintenance decision-making in engineering applications while reducing maintenance costs cut down by 20 percent.

4.2 SWM Model Applicability

The SWM model is applicable to a wide range of scenarios, including:

4.2.1 Monument assessment

By analyzing the wear and tear characteristics of stairs, SWM models analyze stair wear patterns to estimate construction age, historical usage frequency, and maintenance records. By combining material analysis and period-specific techniques, they help optimize restoration timing while preserving authenticity, providing scientific support for cultural heritage conservation and extending structural lifespan.

4.2.2 Modern building maintenance

In modern building management, SWM model analyzes stair wear patterns to optimize maintenance cycles and material selection in buildings. By simulating wear on different materials, they prevent safety risks while reducing costs. The data also guides material choices for new constructions, favoring durable solutions to enhance longevity. This data-driven approach achieves the optimal balance between structural safety and economic efficiency in modern building management.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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