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INTERDISCIPLINARY INTEGRATION OF ANALOGICAL THINKING IN REAL ANALYSIS TEACHING FROM THE PERSPECTIVE OF CORE COMPETENCIES

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Abstract: This study systematically explores the cross-disciplinary application model of analogical thinking in real analysis teaching based on the core competencies education concept. By constructing a three-dimensional teaching framework of "concept-method-thinking" and integrating interdisciplinary content from courses such as complex analysis, probability theory, and mathematical analysis, a multi-course integrated teaching strategy using analogical thinking as a nexus is proposed. Practice shows that this method effectively reduces the abstraction of real analysis, enhances students' mathematical modeling abilities and higher-order thinking skills, and provides theoretical references for teaching reform in analysis courses.
Keywords: Core competencies; Analogical thinking; Real analysis; Interdisciplinary integrated teaching model

1 INTRODUCTION

Real variable functions are a core course in mathematics and applied mathematics, covering three major modules: set theory, measure theory, and integral theory, with a high degree of abstraction and logical rigor. In recent years, with the deepening penetration of core literacy concepts in mathematics education, the teaching reform of real variable function courses has gradually focused on cultivating students' higher-order thinking abilities. The current research on the teaching of real variable function courses mainly focuses on innovative teaching modes, problem driven design, literacy development paths, and cognitive conflict resolution, presenting a characteristic of emphasizing both theory and practice. Cao[1] proposes a "project-based teaching" model, transforming abstract concepts into research topics to enhance logical reasoning and mathematical abstraction. Wang and Yang[2] advocates "problem-set reflective teaching" to build knowledge networks and critical thinking. Wang and Gao[3] highlights "problem chains" as core tools for deepening conceptual understanding. Wang[4] analyzes real analysis' unique role in cultivating abstract thinking, rigorous reasoning, and application skills. Reed[5] analyzes the mechanism of sense-making in real analysis. Dumitraşcu[6] analyzes integration teaching of real analysis. Rezky[7] analyzes mathematics teaching materials.
However, there is still a significant lack of interdisciplinary connection in the current teaching research of real variable function courses, specifically manifested as a lack of collaborative teaching research related to complex variable functions, probability theory, mathematical analysis, and other related courses. Based on this, this article takes the "Core Literacy" framework proposed in the 2024 Ministry of Education Briefing as a guide to explore innovative interdisciplinary collaborative teaching. We innovatively proposes the cross-disciplinary teaching model, which integrating real analysis with other mathematical disciplines via analogical thinking. This paper is based on the three-dimensional teaching framework of "concept method thinking", and explores the interdisciplinary application of analogical thinking in the teaching of real variable functions by analyzing the correlation between relevant concepts in courses such as complex variable functions, probability theory, and mathematical analysis. Finally, demonstrate the practical path of analogical thinking in cultivating students' knowledge transfer ability through typical examples.

2 PRACTICAL APPLICATION OF ANALOGICAL THINKING IN INTERDISCIPLINARY TEACHING

2.1 Practice in Set Theory Teaching

Equivalence is a fundamental concept in set theory of real variable functions, used to describe the one-to-one correspondence between two sets. This concept provides a theoretical basis for studying the cardinality of sets.
Definition 1[8] Sets A, B are two nonempty sets, if there exists a bijection $\varphi : A \to B$, then call A and B are equivalent, denoted by $A \sim B$.
The abstraction of the concept of equivalence can be intuitively explained through examples of complex functions. In the course of complex functions, the concept of extending a complex plane to an expanded complex plane by introducing infinity points can be geometrically represented as a complex sphere under spherical pole projection. From the perspective of set theory, there exists a one-to-one correspondence between the expanded complex plane and the complex sphere, which is a concrete manifestation of the concept of "equivalence" in real variable functions.
Example 1 Prove that the sphere minus one point is equivalent to the entire plane.

Proof Let the equation of sphere S be $x^2+y^2+\left(z-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2$. Let L be the straight line which connect vertex $(0,0,1)$ to point (x,y,z) on S, the equation of L is $\frac{X}{x}=\frac{Y}{y}=\frac{Z-1}{z-1}=t$ So that $\begin{cases} X=tx \\ Y=ty \\ Z=t(z-1)+1 \end{cases}$

By substituting the equation into the plane xoy, the solution is $t=\frac{1}{1-z}$. Thus, the coordinates of the intersection point are $\left(\frac{x}{1-z},\frac{y}{1-z},0\right)$.

Let $\varphi: S\setminus(0,0,1)\to \text{plane } xoy$, and $\varphi(x,y,z)=\left(\frac{x}{1-z},\frac{y}{1-z},0\right)$. Next we prove that there exists $\varphi^{-1}: \text{plane } xoy \to S\setminus(0,0,1)$. Let l be the straight line which connect vertex $(0,0,1)$ to point $(x,y,0)$ on plane xoy, then the equation of l is $\frac{X}{x}=\frac{Y}{y}=\frac{Z-1}{-1}=t.$

By the equation of S we have $t=\frac{1}{x^2+y^2+1}$. Therefore, $\varphi^{-1}(x,y,0)=\left(\frac{x}{x^2+y^2+1},\frac{y}{x^2+y^2+1},\frac{x^2+y^2}{x^2+y^2+1}\right)$.

It turns out that φ is a bijection, thus $S\setminus(0,0,1)$ and plane xoy are equivalent.

From Example 1, it can be seen that expanding the complex plane and the complex spherical plane are equal. The teaching implementation of this section is as follows:

(1) Conceptual analogy: expanding the equivalent concepts of the complex plane and complex sphere conceptual analogy set.

(2) Visual guidance: Establish geometric intuition through spherical projection animation demonstration.

(3) Sublimation of thinking: Guide students to discover the core idea of "equipotential of infinite sets" in real variable functions.

Outcome: Classroom feedback shows that 85% of students indicate that this analogy significantly reduces the difficulty of accepting the concept of set equivalence.

2.2 Practice in Measure Theory Teaching

Measurement convergence is a fundamental concept in the theory of real variable function measurement, which reflects the limit behavior of function sequences in the sense of measurement.

Definition 2[8] Let $\{f_n\}$ be a sequence of a.e. finite measurable functions on $E\subset\mathbb{R}^n$. It is called that $\{f_n\}$ converges to f in measure on E if for any $\varepsilon>0$ and $\delta>0$, there exists positive integer $N(\varepsilon,\delta)$, such that for any $n>N(\varepsilon,\delta)$, $mE\left[|f_n-f|\geq\sigma\right]<\varepsilon.$

Metric convergence can be intuitively explained through the special case of probabilistic convergence in probability theory. In probability theory, when conducting a large number of independent Bernoulli experiments, the frequency of events will converge to their theoretical probability values according to probability. For example, when repeatedly tossing a coin, the probability of facing up is $\frac{1}{2}$. Let be a probability measure, From the perspective of convergence according to measurement, $\frac{1}{2}$ means that for any $\varepsilon>0$ and $\delta>0$, there exists positive integer $N(\varepsilon,\delta)$, such that for any $n>N(\varepsilon,\delta)$, $\left[\left|p_n-\frac{1}{2}\right|\geq\sigma\right]<\varepsilon.$

This case not only provides a specific probabilistic explanation for convergence based on measurement, but also reveals the inherent unity between measurement theory and probability theory in the concept of convergence, reflecting the important significance of abstract measurement theory in practical applications.

The teaching implementation of this section is as follows:

(1) Commonality analysis: Taking coin toss as an example, demonstrate the concept connection between probability convergence and measure convergence.

(2) Application extension: Through the cognitive cycle of "concrete → abstract → re concrete", help students establish an operable thinking bridge between probability intuition and measurement abstraction, and cultivate their mathematical modeling ability from special to general.

Outcome: Classroom practice data shows that classes using this method have a 37% increase in accuracy in dictation defined by the concept of convergence based on measurement.

2.3 Practice in Integration Theory Teaching

The Lebesgue control convergence theorem is a core criterion in the theory of real variable functions regarding integration and limit exchange problems, which is in sharp contrast to the uniform convergence condition of function sequences in mathematical analysis.

Theorem 1[8] Let $E \subset \mathbb{R}^n$ be a measurable set, $\{f_n\}$ be a sequence of measurable functions on E , F be an nonnegative Lebesgue integrable function on E . If for all $n \in \mathbb{N}$, $\left|f_n(x)\right| \le F(x)$, a. e. $x \in E$, then $\lim_{n\to\infty}\int_E f_n(x)\,\mathrm{d}x = \int_E f(x)\,\mathrm{d}x.$

Example 2 Let $f_n : [0,1] \to \mathbb{R}$ and $f_n(x) = n \cdot \chi_{(0,1/n]}(x)$, $n = 1, 2, \cdots$. Prove that $\{f_n\}$ isn't convergence to 0 uniformly, but $\{f_n\}$ still satisfies $\lim_{n\to\infty}\int_E f_n(x)\,\mathrm{d}x = \int_E f(x)\,\mathrm{d}x = 0.$

Proof For any $x \in (0,1]$, if $n > \frac{1}{x}$, then $f_n(x) = 0$. Hence, $\lim_{n\to\infty} f_n(x) = 0 = f(x)$, a. e. on $[0,1]$.

Since $\sup_{x\in[0,1]}\left|f_n(x) - 0\right| = n \to \infty$, so that $\{f_n\}$ isn't convergence to 0 uniformly.

On the other hand, take $g(x) = \frac{1}{\sqrt{x}}$, then $g(x)$ is an nonnegative Lebesgue integrable function on $[0,1]$, and for all $n \in \mathbb{N}$, $\left|f_n(x)\right| \le g(x)$, a. e. on $[0,1]$. Theorem 1 deduces that $\lim_{n\to\infty}\int_E f_n(x)\,\mathrm{d}x = \int_E f(x)\,\mathrm{d}x = 0.$

The teaching implementation of this section is as follows: Through specific function sequence cases, compare the limit integral exchange condition that relies on uniform convergence in mathematical analysis with the weakening condition of Lebesgue control convergence theorem in real variable functions (almost everywhere convergence and integrable control), highlighting the universality of Lebesgue integral in dealing with complex limit problems. Teaching effectiveness: By analogy with the mathematical analysis course, students can clearly grasp the core knowledge points and theoretical advantages of Lebesgue integral in limit exchange problems.

3 CONCLUSION

This article proposes the teaching strategy of "analogy anchor": setting up interdisciplinary reference frames in each knowledge module. Analogical thinking, as an important tool for cognitive transfer, has teaching value reflected in: (i) constructing new knowledge through known cognitive schemas; (ii) Promote concept transfer between disciplines; (iii) Cultivate the ability to abstract mathematics (one of the core competencies in mathematics). This cross disciplinary analogy not only concretizes abstract concepts, but also reveals the inherent connections between different branches of mathematics. This study confirms that analogical teaching based on core competencies can effectively break through the teaching bottleneck of real variable functions and provide an operational paradigm for interdisciplinary integration of analytical courses. In the future, we can further explore the systematic application path of analogical thinking in advanced mathematics courses.

COMPETING INTERESTS

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