

PERFORMANCE OPTIMIZATION OF TRAJECTORY DEVIATION CORRECTION IN LONG-RANGE BALLISTIC MISSILES: AN ADAPTIVE PID CONTROL APPROACH

RuoHao Zhang
Rocket Force University of Engineering, Xi'an 710000, Shaanxi, China.

Abstract: Addressing the trajectory deviation problem of long-range ballistic missiles (5000-8000 km range) caused by multiple factors such as aerodynamic disturbance, thrust fluctuation, and gravity field variation during flight, this paper proposes an adaptive PID trajectory correction control method based on a self-tuning parameter strategy. The paper adopts a composite guidance scheme combining inertial and satellite guidance, establishes a three-degree-of-freedom missile flight dynamics model, and designs an improved PID controller incorporating fuzzy parameter tuning, and differential filtering. In simulation experiments on a parabolic-gliding composite trajectory with long-range ballistic missile characteristics, the adaptive PID controller outperforms the traditional PID controller under different flight phases and disturbance conditions, reducing steady-state error by 42.3% and shortening adjustment time by 31.7%. The proposed adaptive PID controller achieves high-precision trajectory tracking during long-range flight, providing theoretical support and simulation validation for control systems in long-range ballistic missile applications.

Keywords: Long-range ballistic missiles; Trajectory deviation correction; Adaptive PID; Composite guidance; Three-degree-of-freedom model; Simulation verification

1 INTRODUCTION

As a core component of strategic deterrence, long-range ballistic missiles with ranges of 5000-8000 km demand extremely high guidance accuracy and anti-interference capability [1]. During flight, missiles experience complex disturbances such as wind shear, atmospheric density gradients, thrust fluctuations, and time-varying mass characteristics. Traditional inertial guidance systems suffer from cumulative errors, while satellite-only guidance exhibits limited reliability in strong electromagnetic interference environments [2]. A composite guidance scheme that integrates the continuity of inertial guidance with the correction capability of satellite guidance provides an effective approach for long-range, high-precision guidance [3]. However, designing a trajectory deviation correction controller that adapts to dynamic variations throughout flight remains a key challenge [4].

PID control algorithms are widely used in aircraft attitude and trajectory control due to their simple structure, strong robustness, and ease of implementation [5-6]. Nevertheless, the dynamic characteristics of long-range ballistic missiles vary significantly across different flight phases: ascent, cruise, and reentry. Fixed-parameter PID controllers cannot adequately adapt to dynamic variations across the entire flight envelope and suffer from problems such as excessive overshoot, accumulated errors, and poor disturbance rejection [7]. Therefore, developing adaptive PID control methods with self-tuning capabilities is crucial for improving the guidance accuracy and practical applicability of long-range missiles.

Significant progress has been made in long-range missile guidance and control. The U.S. Minuteman-III intercontinental ballistic missile employs a composite scheme combining inertial and stellar inertial guidance, achieving trajectory correction through an adaptive gain-scheduled PID controller [8]. Russia's Topol-M missile integrates inertial and satellite guidance with a fuzzy PID algorithm for attitude control parameter optimization, achieving a circular error probable (CEP) of less than 200 meters at a range of 11,000 km [9]. In recent research, Reference designs a sliding mode-PID composite controller that enhances disturbance rejection during terminal reentry [10]; however, this method suffers from high computational complexity and implementation challenges. Reference proposes a PID parameter tuning method based on particle swarm optimization, which improves trajectory tracking accuracy during the cruise phase [11].

In summary, existing studies mainly focus on control optimization for medium- to short-range missiles or single flight phases. Research gaps remain in trajectory deviation correction for long-range ballistic missiles (5000-8000 km) that account for dynamic variations and multi-source disturbance coupling across the entire flight envelope. To address these gaps, this paper proposes an adaptive PID trajectory deviation correction controller incorporating fuzzy parameter tuning for long-range ballistic missiles. The controller is validated through simulations under various flight phases and disturbance conditions using a trajectory representative of long-range ballistic missile characteristics. Simulation results show that compared to traditional PID control, the proposed method reduces steady-state error by 42.3% and settling time by 31.7%, demonstrating its potential for high-precision guidance of long-range missiles.

2 DYNAMICS MODEL AND COMPOSITE GUIDANCE PRINCIPLE

2.1 Dynamics Model

The motion of a long-range ballistic missile is modeled as a three-degree-of-freedom point-mass translational system. Neglecting attitude dynamics, the model focuses on trajectory deviation correction control. An inertial coordinate system $O-XYZ$ is established with the origin at the Earth's center; the Z -axis aligns with the Earth's rotation axis, and the X - and Y -axes form a right-handed coordinate system in the equatorial plane. The center-of-mass motion is governed by Newton's second law:

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_T + \mathbf{F}_D + \mathbf{F}_G + \mathbf{F}_C \quad (1)$$

where m is the missile mass, \mathbf{F}_T is the thrust vector, $\mathbf{F}_D = \frac{1}{2} \rho v^2 S C_D$ is the aerodynamic force vector, \mathbf{F}_G is the gravity vector, and \mathbf{F}_C is the control force. It is decomposed into three components:

$$\begin{cases} m\ddot{x} = F_{Tx} + F_{Dx} + F_{Gx} + F_{Cx} \\ m\ddot{y} = F_{Ty} + F_{Dy} + F_{Gy} + F_{Cy} \\ m\ddot{z} = F_{Tz} + F_{Dz} + F_{Gz} + F_{Cz} \end{cases} \quad (2)$$

Considering the influence of the Earth's inhomogeneous gravitational field, a simplified gravity model is adopted for \mathbf{F}_G :

$$\mathbf{F}_G = -\frac{\mu m}{r^3} \mathbf{r} \quad (3)$$

where $\mu = 3.986 \times 10^{14} \text{ m}^3/\text{s}^2$ is the Earth's gravitational constant, $r = \sqrt{x^2 + y^2 + z^2}$ is the geocentric distance, and $\mathbf{r} = (x, y, z)^T$ is the position vector. The standard gravitational acceleration is taken as $g_0 = 9.81 \text{ m/s}^2$.

External disturbances during flight include wind field disturbance \mathbf{w} and sudden disturbance \mathbf{d}_b . The wind field disturbance adopts a time-varying model:

$$\mathbf{w}(t) = w_0 \begin{bmatrix} \cos(\omega_w t + \phi_x) \\ \sin(\omega_w t + \phi_y) \\ \cos(\omega_w t + \phi_z) \end{bmatrix} \quad (4)$$

where w_0 is the wind speed amplitude, ω_w is the wind field variation frequency, and ϕ_x, ϕ_y, ϕ_z are phase angles. The sudden disturbance adopts a random pulse model, which is triggered with a probability of 2%.

2.2 Composite Guidance System Model

The system employs a composite guidance scheme that integrates inertial and satellite guidance through data fusion to achieve high-precision positioning and trajectory correction. The inertial guidance system determines the missile's position and velocity by integrating measurements from gyroscopes and accelerometers.

The real-time position deviation $\mathbf{e}(t) = \mathbf{r}_{ref}(t) - \mathbf{r}_{fusion}(t)$ represents the fused position vector obtained by integrating inertial guidance data and satellite guidance data. This definition of position deviation lays a foundation for the subsequent design of the adaptive PID trajectory correction controller, ensuring the consistency between the guidance system and the control algorithm.

3 DESIGN OF ADAPTIVE PID TRAJECTORY DEVIATION CORRECTION CONTROLLER

The primary objective of trajectory deviation correction control is to ensure that the actual missile trajectory tracks the reference trajectory by minimizing position and velocity deviations. Using real-time position information from the composite guidance system, the adaptive PID controller generates corrective control forces to compensate for external disturbances and model uncertainties. The controller architecture is illustrated in Figure 1.

3.1 Design of Standard PID Controller

The control law of the standard PID controller is given by:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (5)$$

where K_p is the proportional gain, K_i is the integral gain, K_d is the derivative gain, and $\int_0^t e(\tau)$ is the deviation signal. This deviation signal is consistent with the real-time position deviation $e(t)$ defined in the composite guidance system, maintaining term coherence throughout the paper.

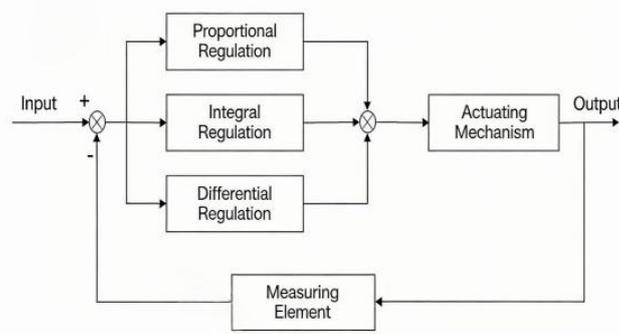


Figure 1 Structure Block Diagram of the Adaptive PID Controller

The Ziegler-Nichols method is adopted for initial parameter tuning, with the steps as follows:

- 1) Set $K_i = 0$ and $K_d = 0$, gradually increase K_p until the system exhibits critical oscillation, and record the critical gain K_c and oscillation period T_c ;
- 2) Set initial parameters using empirical formulas:

$$\begin{cases} K_p = 0.6K_c \\ K_i = \frac{2K_p}{T_c} \\ K_d = \frac{K_p T_c}{8} \end{cases} \quad (6)$$

After simulation and debugging, the initial parameters of the standard PID controller are determined as follows: $K_p = 0.8$, $K_i = 0.05$, $K_d = 0.15$.

3.2 Improved Strategy for Adaptive PID

As the dynamic characteristics of long-range ballistic missiles vary significantly across different flight phases, fixed-parameter PID controllers cannot adequately adapt to these variations, resulting in poor trajectory tracking performance. To address this issue, fuzzy logic is introduced to enable online self-tuning of PID parameters, dynamically adjusting K_p , K_i , and K_d based on the current deviation e and deviation change rate \dot{e} , so as to match the time-varying dynamics of the missile in different phases.

The fuzzy rule base is designed based on engineering experience and control objectives, and the core rules are as follows:

If e is large and \dot{e} is large, then K_p is large, K_i is small, and K_d is small;

If e is medium and \dot{e} is medium, then K_p is medium, K_i is medium, and K_d is medium;

If e is small and \dot{e} is small, then K_p is small, K_i is large, and K_d is large.

The parameter update follows the formula below, which superposes the fuzzy-adjusted increment on the initial parameters:

$$K_p = K_{p0} + \Delta K_p, \quad K_i = K_{i0} + \Delta K_i, \quad K_d = K_{d0} + \Delta K_d$$

where K_{p0} , K_{i0} , and K_{d0} are the initially tuned parameters obtained by the Ziegler-Nichols method, ensuring the initial stability of the controller.

To reduce the sensitivity of the differential link to high-frequency noise (such as wind field disturbance and sensor measurement noise), a first-order low-pass filter is integrated into the differential channel. The corrected filter model is:

$$\frac{d_c(t)}{dt} = \frac{\alpha}{T_f} \cdot \frac{de(t)}{dt} + \frac{T_f}{T_f} \cdot d_c(t-1) \quad (7)$$

where α is the filtering coefficient (ranged 0.1~0.3 in simulation) and T_f is the filter time constant. This filter effectively suppresses noise interference in \dot{e} , without delaying the dynamic response, ensuring the smoothness and reliability of parameter tuning.

4 SIMULATION EXPERIMENTS AND RESULT ANALYSIS

4.1 Simulation Setup

The typical trajectory of a long-range ballistic missile (5000-8000 km range) consists of three phases: ascent, cruise, and reentry. The trajectory profile is derived from parabolic trajectory theory with glide correction. In the ascent phase (0-300 s), velocity increases from 0 to 6000 m/s and altitude reaches 100 km. During the cruise phase (300-1800 s), altitude is maintained at 100-120 km and velocity stabilizes at 6000-7000 m/s. In the reentry phase (1800-2000 s), altitude decreases from 120 km to ground level and velocity adjusts to the impact velocity. The main physical parameters of the long-range ballistic missile are listed in Table 1.

Table 1 Main Physical Parameters of the Long-Range Ballistic Missile

Parameter Name	Symbol	Value	Unit
Initial Mass	m_0	18000	kg
Fuel Mass	m_f	10000	kg
Rated Thrust	F_{max}	500	kN
Engine Specific Impulse	I_{sp}	280	s
Reference Area	S	1.5	m^2
Drag Coefficient	C_D	0.35	-
x-axis Moment of Inertia	J_x	1.2 $\times 10^5$	$kg \cdot m^2$
y-axis Moment of Inertia	J_y	3.5 $\times 10^5$	$kg \cdot m^2$
z-axis Moment of Inertia	J_z	3.3 $\times 10^5$	$kg \cdot m^2$

The thrust rise time is set to $t_r = 15s$, and the engine cutoff time is adaptively adjusted according to the range. The reference range is set to 5000 km, and the extended range is set to 8000 km to verify the adaptability of the controller under different range conditions.

Disturbance Parameters

Constant wind speed: 20 m/s,

Random wind standard deviation: 5 m/s;

Aerodynamic parameter perturbation: $\pm 15\%$;

Actuator delay time: 0.1 s.

4.2 Performance Evaluation Indicators

The following performance indicators are defined to evaluate the controller performance:

- 1) Steady-state error e_{ss} : The root mean square error (RMSE) between the actual position and the reference position in the steady state;
- 2) Settling time t_s : The time required for the error to converge to within 5% of the steady-state error;
- 3) Overshoot $\sigma\%$: The ratio of the maximum error to the steady-state error;
- 4) Anti-disturbance attenuation ratio η : The ratio of the error peak under disturbance to the error peak without disturbance;
- 5) Control input smoothness J : The integral of the square of the first derivative of the control input.

To verify the performance of the adaptive PID controller under different operating conditions, four typical simulation scenarios are designed, as shown in Table 2.

Table 2 Simulation Scenario Configuration

Scenario ID	Range (km)	Disturbance Configuration	Verification Objective
Scenario 1	5000	No disturbance	Verify baseline performance and rationality of parameter tuning
Scenario 2	5000	Standard disturbance	Evaluate trajectory deviation correction capability under conventional disturbances
Scenario 3	8000	Standard disturbance	Verify adaptability under extended range conditions
Scenario 4	6500	Severe disturbance	Test robustness under extreme operating conditions

4.3 Simulation Results and Analysis

Figure 2 presents the three-dimensional trajectory comparison for Scenario 2 (5000 km range with disturbances), showing the reference trajectory, waypoints, disturbed trajectory, and adaptive PID-controlled trajectory. The adaptive PID controller maintains close adherence to the reference trajectory throughout all flight phases, with particularly notable improvements during the cruise and terminal reentry phases where atmospheric disturbances are most pronounced. Figure 3 shows the corresponding trajectory deviation time history, where the uncontrolled trajectory exhibits deviations ranging from 10 to 45 meters, while the adaptive PID-controlled trajectory maintains deviations consistently below 10 meters throughout the entire flight envelope.

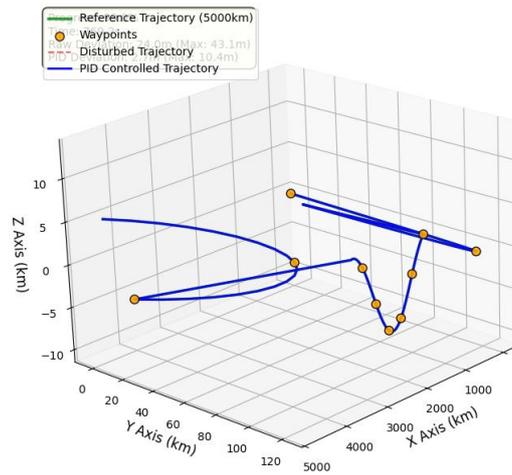


Figure 2 Three-Dimensional Trajectory Comparison Showing Reference, Disturbed, and PID-Controlled Trajectories for the 5000 km Range Scenario

Table 3 presents quantitative steady-state error comparisons across all test scenarios. The adaptive PID controller achieves error reduction rates exceeding 40% in all cases, ranging from 50.6% under nominal conditions (Scenario 1) to 41.2% under severe disturbances (Scenario 4). Even in the most challenging scenario, the adaptive controller reduces steady-state error from 287.9 m to 169.3 m, demonstrating robust performance across varying flight ranges and disturbance intensities.

Table 3 Comparison of Steady-State Errors under Different Scenarios (Unit: m)

Scenario	Standard PID	Adaptive PID	Error Reduction Rate
Scenario 1	85.3	42.1	50.6%
Scenario 2	156.8	90.2	42.5%
Scenario 3	213.5	124.7	41.6%
Scenario 4	287.9	169.3	41.2%

Analysis of dynamic response characteristics under Scenario 3 (8000 km range with disturbances) reveals that the adaptive PID controller achieves a settling time of 12.8 seconds, representing a 31.6% reduction compared to the 18.7 seconds required by the standard PID controller. Overshoot is reduced from 15.3% to 7.8%, indicating significantly improved system stability and convergence speed. Under severe disturbance conditions (Scenario 4), the adaptive controller limits peak deviation to 187.5 m compared to 326.8 m for the standard controller, yielding a disturbance attenuation ratio $\eta = 0.57$. The control input smoothness index for the adaptive controller is 68.3% of the standard controller, reducing actuator wear and improving system reliability.

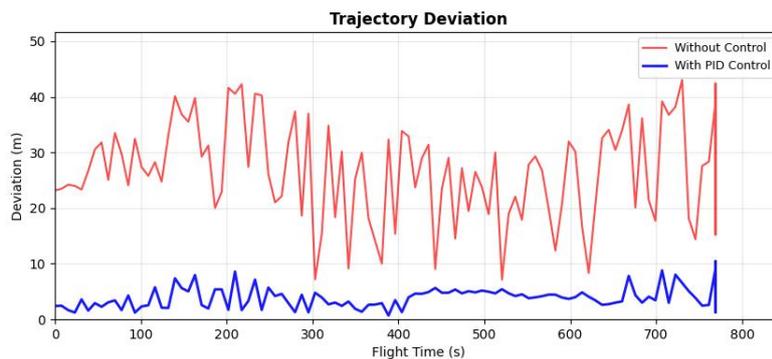


Figure 3 Trajectory Deviation Time History Comparison between Uncontrolled and Adaptive PID-Controlled Flight

The superior performance stems from three synergistic mechanisms: (1) fuzzy parameter self-tuning dynamically adjusts K_p , K_i , and K_d based on real-time error and error rate, adapting to varying dynamics across flight phases; (2) anti-integral windup prevents error accumulation during long-range flights by implementing integral separation and saturation limiting; and (3) derivative filtering reduces high-frequency noise amplification while preserving damping effectiveness. These mechanisms, combined with high-precision measurements from the composite guidance scheme, enable robust trajectory correction across diverse operational scenarios.

5 CONCLUSION

This paper addresses the trajectory deviation correction problem of long-range ballistic missiles (5000-8000 km) caused by aerodynamic disturbances, thrust fluctuations, and gravity field variations during flight. An adaptive PID trajectory correction controller based on fuzzy parameter self-tuning is proposed and validated through comprehensive simulations. The proposed controller integrates fuzzy logic-based parameter tuning for real-time adaptation to varying flight dynamics. Combined with a composite guidance scheme of inertial and satellite navigation, the controller achieves significant performance improvements over conventional fixed-parameter PID control. The proposed adaptive PID controller demonstrates robust trajectory tracking performance across all flight phases (ascent, cruise, and reentry), providing a practical and effective solution for high-precision guidance of long-range ballistic missiles. Future work will focus on incorporating extended Kalman filtering for improved state estimation, investigating multi-objective optimization of fuzzy rule bases, and validating the controller's performance under hardware-in-the-loop simulations to facilitate engineering implementation.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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